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RESEARCH

A New Experimental Approach to Weight Change Experiments at the Moment of Death with a Review of Lewis E. Hollander's Experiments on Sheep

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Abstract—A critical review is conducted based on analytical simulations of an experimental study to measure change in weight of sheep upon death published in 2001 by L. E. Hollander in *JSE*. The experimental system is modeled as a single-degree-of-freedom vibrating system. The following conclusions are obtained. (1) The experimental result obtained with Sheep #7 appears to be natural, as expected by the theoretical model. (2) Hollander's conclusion that "there was a transient gain of weight of 780 grams" in the case of Sheep #7 is not an appropriate expression of the experimental result, because the 780 gf pulse includes an overshoot reaction of the system; however, the cause of the force event remains to be explained. Analytical simulation of a supposed weight measurement experiment involving an out-of-body experience (OBE) subject is carried out using the theoretical model under a supposed weight decrease of the experient. The simulation showed that the disturbance caused by breathing becomes the primary noise in the system response. However, some noise reduction techniques can be used to discern the change in the weight of the experient, if there indeed is a weight decrease. Weight measurement experiments using a trance channeler are suggested because "trance channeling" is objectively more observable than OBE.

Keywords: critical review—analytical simulation of experiment—transient weight gains—death of sheep—analytical model of vibration—overshoot reaction—disturbances due to cardiac and breathing activities—simulation of OBE—supposed weight decrease during OBE—suggested experiments with trance channeler

1. Introduction and Objectives

It has been a little over 100 years since the paper by Duncan MacDougall, MD, was published concerning an experimental study of the change in the weight of the human body in life-to-death transition [1]. Since then, there have been several skeptical as well as critical arguments against MacDougall's paper, specifically those expressed in books by a psychologist (Susan Blackmore, 1982) [2] and a scientist (Len Fisher, 2004) [3]. Similar skeptical arguments are posted on many

Web sites. Most of these arguments are similar, stating how "his experiment was sloppy; his claimed weight of the soul turned out to be simply the result of sloppy science; his experiment was silly, you'd need not just a scale, but a completely isolated system." From a scientific point of view, it can be shown with relative ease that none of these criticisms have a quantitative basis. For example, Len Fisher's speculation [3] of "convection currents" of air to explain the missing 21 g requires an updraft ranging from 40 to 55 cm/s against the whole flat bottom area (assuming that it is in the range from 2 to 1 m^2, respectively) of MacDougall's cot bed on the scale platform (this can be easily shown based on a stagnation-point flow model using the Bernoulli equation [4]). Inducing such an air velocity of a natural-convection updraft requires, for example, an array of heated vertical plates with a height of 1 ft covering the entire flat bottom area with a temperature that exceeds the ambient air temperature by more than 90°C (experimental data can be found in McAdams [5]), depending upon the shape and size of the heated plates. Contrary to this thermo-hydraulic reasoning, Fisher speculates that the convection currents may be induced by (not an increase, but) a "decrease" in the patient's body temperature upon death. Indeed, it will be very difficult to scientifically refute the missing weights in MacDougall's experiment, even though his experiment, conducted around 100 years ago, may appear sloppy from the viewpoint of today's scientific standards.

Apparently, Lewis Hollander's paper published in the *Journal of Scientific Exploration* [6] was stimulated by the 100-year-old MacDougall paper. Although the author writes that the study is very much preliminary, it is felt that a technical review of his experimental results is necessary, and this is the primary motivation for the present paper. In addition, in this paper, an analytical simulation of the probable responses of Hollander's weighing system will be shown in a supposed weight measurement experiment of a subject during his/her out-of-body experiences (OBEs) to understand the difficulties, if any, in such experiments.

2. Trial Simulation of Hollander's Experiment

2.1. Characteristic Parameters of Weighing System

One of the best ways to understand the experimental results is to carry out a simulation of the experiment by using a simple analytical model for the experimental weighing system. Although Hollander's paper gives very little information on the experimental system, a mathematical model of vibration based on a single degree of freedom can be created for the weighing system using data available from the paper. The basic equation of the model of damped vibration for a mass "m" under an externally applied force can be expressed as follows (see standard text books on physics or vibration engineering, e.g., [7]):

$$x'' + 2\sigma x' + \omega_n^2 x = F(t)/m, \tag{1}$$

where

x = small displacement (in meters) of the mass from its equilibrium position ($x = 0$); x'' and x' are the time derivatives of x, i.e., acceleration and velocity, respectively; the positive direction of "x" is defined here as vertically downward, along the direction of gravity;

σ = vibration decay rate (1/s);

$\omega_n(=k_{eq}/m)^{0.5}$ = natural angular frequency of the system (rad/s);

k_{eq} = equivalent spring constant of the system (N/m);

$F(t)$ = time (t) dependent external force applied to the system, expressed as $F_o \times f(t)$ with dimensionless function f(t) and normalization force factor F_o (N);

m = mass (kg) of the system, which is supposed to be constant throughout the experiment.

This mathematical model is intended to predict only the small vibration behavior of the mass with respect to its equilibrium position by an action of an external force applied to the system. The model is not intended to predict the change in the weight of the system. If there is a small change in the mass "m," the effect may be expressed as an external force F(t) that simulates the removal or addition of the corresponding load. The assumption of constancy of mass "m" above is an approximation of the model; in other words, a small decrease (Δm) in "m" (due to the loss of moisture evaporating from the animal bodies, as reported in Hollander [6]), in comparison to the initial mass (m_0), shall not considerably affect the vibration behavior of the weighing system. In the experiment, Δm was less than 0.1% of the initial mass, m_0. The experimental quantity "Weight in Kilograms" expressed in the ordinate of the figures in Hollander's paper may be related to the "x(t)" of Equation 1 as follows:

$$k_{eq} \times x(t) = \text{change in weight at time "t" from its equilibrium weight.}$$

The "Weight in Kilograms" expressed in the ordinate of, for example, Figure 2 of Hollander will be equated to "$k_{eq} \times x(t)$" + equilibrium weight of the system at time "t." (Here, the term "equilibrium" indicates that the system is not in motion.) But this reasoning is scientifically wrong unless the mass of the system is not in motion at time "t." Hence, the physical quantity "$k_{eq} \times x(t)$" will be denoted as a "system response," which can still be compared with the experimental vibration behaviors shown in the figures in Hollander [6]. The characteristic parameters of system vibration will be estimated in the subsequent sections.

(1) Weight of system (M_p) without animal subject. Hollander reports that the system consists of a platform (size: 215 × 92 cm) on a steel frame, which is set on four load cells of 45-kg capacity each; hence, the maximum allowable load on the load cells will be 180 kg. According to the paper, "the full-scale capacity of the system was 100 kg, with a sensitivity of −5 gm"; from this, the total weight of the steel frame and platform without an experimental animal subject on the platform is assumed to be $M_p = 80$ kg (maximum). The total mass of the system

"m" in Equation 1 becomes "$M_p + m_{sp}$," where m_{sp} is the mass of the experimental animal subject on the platform. It was reported in the paper that "the measured response time of the system was 0.2 seconds," which may be taken to indicate that any details of a force event occurring within 0.2 s will not be reliably recorded by the weighing system.

(2) Characteristic parameters of vibration of system. The natural angular frequency of the system (ω_n) with experimental subject Sheep #7 ($m_{sp} = 70.2$ kg) on the platform will be estimated from the damped free vibration behavior of the system shown in Hollander's Figure 2. The vibration pattern is shown in the figure with a period of about $T_d = 1.9$ s during the time period from about 56 to 71 s during transience, which gives the angular frequency of damped free vibration, $\omega_d (= 2\pi/T_d) = 3.31$ rad/s.

This damped vibration behavior also gives the vibration decay rate "σ" of the system. The logarithmic decrease in the damped vibration, $\delta = \ln(x_{n-1}/x_n)$, will be estimated as 0.263 from the figure by fitting six to seven x_n data points, where x_n's are the peak amplitudes of successive damped vibrations from the equilibrium value. Using the relationships $\delta = \omega_n \times \zeta \times T_d$ and $\omega_d = \omega_n \times (1 - \zeta^2)^{0.5}$, where ζ is the viscous damping factor, these parameters can be estimated to be

$$\omega_n = 3.31 \text{ (rad/s)},$$

$$\zeta = 0.042 \text{ (–), and}$$

$$\sigma = \omega_n \times \zeta = 0.138 \text{ (1/s)}.$$

ω_n is almost equal to ω_d because of the small ζ value. These parameters are specific to the case of the experiment with Sheep #7. The angular frequency, ω_n, is related to the equivalent "spring constant" of the system as follows:

$$\omega_n^2 = k_{eq}/(M_p + m_{sp}) = (k_{eq}/M_p)/(1 + m_{sp}/M_p),$$

where

k_{eq} = equivalent spring constant of the system (N/m),
M_p = mass of the system as defined above (=80.0 kg),
m_{sp} = mass of the experimental subject (kg).

The equivalent spring constant is calculated as $k_{eq} = M_p \times \omega_n^2 \times (1 + m_{sp}/M_p) = 1.646 \times 10^3$ N/m. The natural angular frequency (ω_o) of the system without an experimental subject on the platform and ω_n (with an experimental subject on the platform) are related to each other as follows:

$$\omega_o = (k_{eq}/M_p)^{0.5},$$
$$\omega_n = \omega_o/(1 + m_{sp}/M_p)^{0.5}. \quad (2)$$

Then ω_o becomes 4.54 rad/s.

Although these characteristic parameters (k_{eq}, σ, ζ) of the natural vibration of the system have been estimated from the result of the case of Sheep #7, it is assumed that these are also applicable to other experimental cases. These characteristic parameters of the weighing system can be and should be determined experimentally first by disturbing the system using an inert mass instead of placing a live animal on the platform. These parameters provide essential information when the experimental results are interpreted.

The linear differential Equation 1 can be solved numerically by using the Euler-Romberg method after expressing the equation in a non-dimensional form by introducing non-dimensional time "τ" ($\tau \equiv t/T_o$, $T_o \equiv 2\pi/\omega_n$) and displacement "X" ($X \equiv x/x_o$, $x_o \equiv T_o^2 \times F_o/m$). A computer program (with double precision in FORTRAN 77 on a PC) for the numerical solution has been created for this study. The program has been validated by comparing its numerical solutions with the analytical solutions for several sample problems easily available in text books of physics or vibration engineering. In what follows, numerical simulations of several supposed problems are carried out to elucidate the significances of Hollander's experiment.

2.2. *Simulation of "Missing 21 Grams" with Weighing System*

First, let us suppose that the first case of MacDougall's experiment is conducted using Hollander's system. According to MacDougall's paper [1], the loss of 21 g on his platform scale was observed "in a few seconds" after the judgment of patient death. Here, let us suppose the following three simplified modes of weight decrease in "the few seconds":

(a) instantaneous decrease,
(b) decrease in 1.5 s at constant rate, and
(c) decrease in 3 s at constant rate.

The initial weight of the patient is arbitrarily assumed to be 62.0 kg (which is the present author's weight). The system is supposed to initially be at an equilibrium state without motion, which means that any disturbance caused by the live patient before death is neglected. The time of death is supposed to be 5 s into the transient for this calculation. F_o in Equation 1 becomes −0.2058 N (which is −21 gf) and f(t) is determined for each mode of the weight decrease history. The calculated system responses to the three modes of weight decrease are shown in Figure 1 as a function of time.

All three cases finally stabilize at a loss of 21 g in about 35 s since the start of the decrease. The system, however, responds differently depending upon the rate of decrease. The vibration of the weighing system (or "ringing" of the load cells) is largest in the case of an instantaneous decrease, in which case, of course, we do not say that there was a "transient weight loss" of 40 g initially in the transient. The large swing beyond 21 g is just an overshoot of the system responding to the fast loss of the small load. If the decrease occurs slowly, the overshoot will become small, as in the case of mode (c). Regardless of the mode of the weight decrease, the system settles in the same new equilibrium state with the system mass decreasing by 21 g from the initial state.

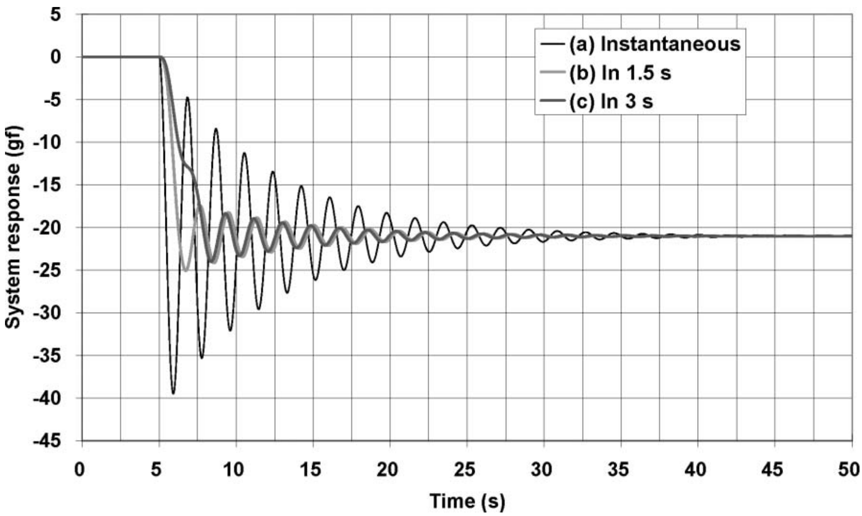


Fig. 1. System responses to three modes of 21 g decrease.

This parametric case has been chosen to show that Hollander's writing of "weight gain transient of 780 grams for 4 seconds" in the case of Sheep #7 in his paper [6] is not an appropriate expression; the "weight gain transient" includes some overshoot vibration of the system! The mentioned "transient gain of 780 grams" in his Figure 2 apparently shows only a small undershoot of the system, and this might have led to Hollander's expression. However, we can make a case with no undershoot and a case with small undershoot after a large single pulse, in which a square impulse of an external force is imposed on a system with time width $\Delta T = 2\pi/\omega_n = 1.897$ s and vibration decay rate $\sigma = 0.0$ and 0.138/s, respectively. These are considered to be parametric cases of simulation of Hollander's experiment, as discussed in Sec. 2.3 with Sheep #7 with an initial weight of 70.2 kg.

Figure 2 shows the system responses with and without damping to the imposed square impulse. In the case without damping ($\sigma = 0$), the impulse of height F_o (415.4 gf) with width $\Delta T = 2\pi/\omega_n$ gives a peak pulse height of 831 gf, which is exactly two times the impulse height F_o. This case has the analytical solution to the problem (with the start of impulse at t = 0), the peak amplitude of which after impulse ($t > \Delta T = 2\pi/\omega_n$) is proportional to $\sin(\omega_n \Delta T/2)$; hence, the amplitude becomes zero for $\Delta T = 2\pi/\omega_n$. During the impulse ($0 < t < \Delta T = 2\pi/\omega_n$), the analytical solution is given as $F_o \times (1 - \cos[\omega_n t])$; hence, the peak height becomes $2 \times F_o$ at $t = \Delta T/2 = \pi/\omega_n$. One may wonder why the mechanical work done by giving impulse to the system disappears after the impulse, $t > \Delta T = 2\pi/\omega_n$. Actually, the positive work (which is defined as $\int F_o dx$, with displacement "dx[t]" in the positive direction) performed by the impulse during

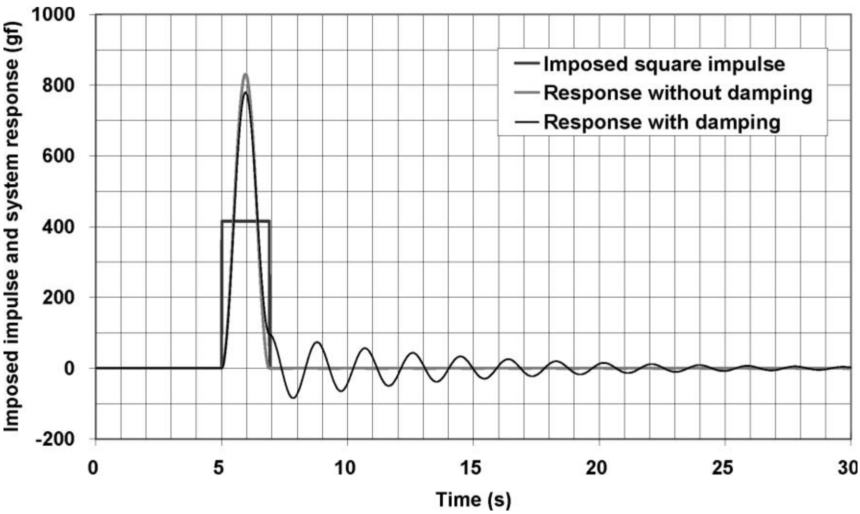


Fig. 2. System response to imposed square impulse with or without damping.

the first half of the impulse is canceled out by the negative work (due to displacement "dx[t]" in the negative direction, i.e., deceleration of motion) performed during the last half of the impulse, and the system settles in equilibrium of no motion at the end of the impulse. In the case with damped vibration ($\sigma = 0.138/$s), the peak height becomes 780 gf (with the intentionally specified impulse height $F_o = 415.4$ gf). Because of damping (i.e., energy dissipation), the maximum height of the peak becomes less than $2 \times F_o$. The amplitude of the vibration after the impulse is relatively small because of the specified impulse width $\Delta T = 2\pi/\omega_n$. These system responses have nothing to do with "contradiction" to or "violation" of Newton's third law of motion (as mentioned in [6]); they are the results of Newton's three laws of motion.

Although these cases have been intentionally created, the results show an overshoot to 780 gf or more under the action of a square impulse of 415.4 gf (4.073 N) height. Hence, we cannot say that there was a "weight gain transient of 780 grams" in the case with Sheep #7 in Hollander's experiment. The actual impulse height that caused the 780 gf might have been only 415 gf in the experiment, the cause of which is yet to be determined. It is not a "weight gain transient of 780 grams," but a "force impulse giving a transient pulse of 780 gram-force" in the system response. Any change in weight can be concluded only when the weighing system has settled in an equilibrium state, although this is an idealized condition. Further, although the width of the 780 gf pulse is depicted as 4 s in Hollander's Figure 2, it appears to be only about 2.6 s. Which of these values is correct? The question can similarly be posed for his Figure 3.

2.3. Simulation of Hollander's Experiment for Interpretation of Results

Many questions can be raised about the experimental results reported in Hollander [6]. Some of them are as follows:

(a) Why did the typical vibration behavior recorded in the case with Sheep #7 (in his Figure 2), apart from its amplitude, not appear in the cases with Sheep #3 and #8 (in his Figures 1 and 3, respectively)? What caused the strange vibration pattern in the Sheep #8 case with 21 data points sampled per second as in the Sheep #7 case? Did the system function normally in the two cases with Sheep #3 and #8?
(b) What was the cause of the 780 gf peak pulse in the Sheep #7 case?
(c) Can we see any effects of breathing and cardiac activities on the system response in the case of Sheep #7?

These questions will be addressed in the following simulations.

2.3.1. Simulation of Case with Sheep #7

(1) Events that affect weighing system. Any movement, whether visible externally or not, of the experimental subject on the platform will affect the weighing system. Breathing and cardiac activities as external forces acting on the system may be specified by the following parameters:

(a) angular frequency, ω_{ext}, and
(b) amplitude of disturbances.

According to the paper [6], the cardiac frequency of the subjects changed transiently from a normal value of 70 to a rapid crisis value of 120 beats per minute, values that correspond to frequencies of $f = 1.167$ and 2.0 Hz, respectively. These frequencies correspond to an angular frequency $\omega_c = 2\pi \times f$ of 7.3 and 12.5 rad/s, respectively, both of which are more than two times higher than the natural vibration frequency of the system, $\omega_n = 3.31$ rad/s. The paper provides no information on the breathing frequency of the subjects. According to biological data presented in a science handbook [8], the normal breathing frequency of sheep is in the range of 16 to 24 breaths per minute, which gives an angular frequency ranging from 1.7 to 2.5 rad/s. If the breathing frequency of Sheep #7 at crisis is assumed to be 30 breaths per minute, the angular frequency becomes $\omega_b = 3.14$ rad/s, which is very close to the natural frequency of the system. These angular frequencies of cardiac activity and breathing indicate that in the experiments, breathing activity might have affected the system response much more than cardiac activity.

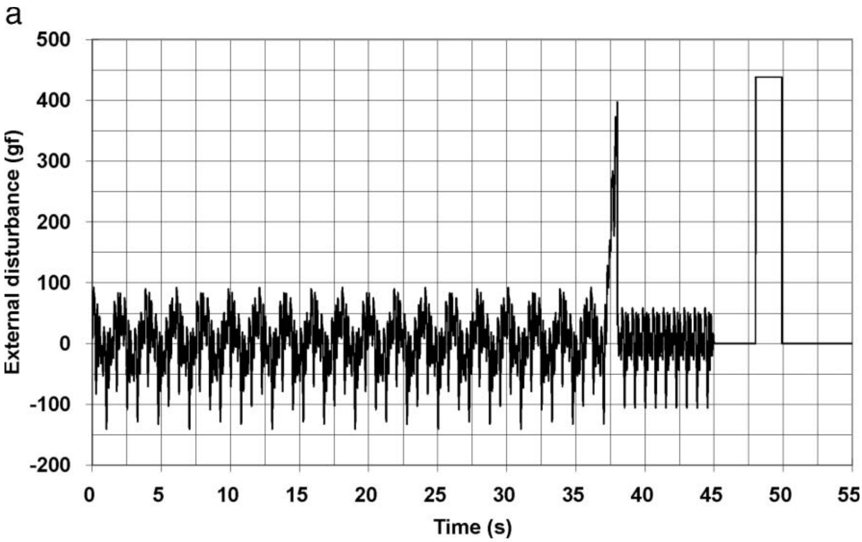
As regards the amplitudes (F_o) of these disturbances as external force $F_o \times f(t)$, we do not have much information at hand. The amplitudes will be treated as parameters in the simulations. Hollander [6] writes in the "Discussion" of his paper that "The normal breathing appears as a rhythmic series of inertial weight gains followed by corresponding losses," which may suggest that "the inertial weight gains and losses" are due to air mass inhaled and exhaled by the lungs of sheep. If this is the intended meaning of the author, however, it is not correct, because the amplitude of vibrations (about 300 gf in the case of Sheep #7) is far greater than the effect of the change in air mass. The inhaled/exhaled mass of atmospheric air per "normal" breath of sheep never exceeds 1 g (it may be about 0.6 g at most based on the air volume per breath, as given in the science handbook [8]; see Table A1 in the Appendix). The disturbances in Hollander's Figure 2 persisted up to the last breath of Sheep #7, and they might have been caused by body sway accompanied with breathing in crisis.

According to Hollander (Figure 2) (and others), there might have been a remarkable disturbance during the last breath of the subjects. This disturbance may be simulated with a triangular impulse with a width of 1 second at the bottom. The amplitude will be treated as a parameter. The last sporadic disturbance after the last breath of the subject will be simulated with a square impulse, as discussed in Sec. 2.2. It is assumed in the calculation that the weighing system is initially at equilibrium state without motion. The reported evaporation of moisture from the animal subject during the experiment is not included in the simulation; hence, the simulation is intended only to observe the vibration behavior of the weighing system with an experimental subject of constant weight on the platform.

The time step size for the numerical solution is constant, about 7 ms, although the Euler-Romberg algorithm automatically cuts down the size until a required convergence criterion is satisfied. The convergence criterion used for the change of solutions (for dimensionless displacement and velocity) in successive iterations is 10^{-7}, which corresponds to a convergence in 0.01% for displacement (x) and in less than 0.01% for velocity (x′).

(2) Results of simulation. A simulation of the experiment with Sheep #7 is shown in Figure 3a and b. In the calculation, the cardiac vibration effect was modeled by using the vertical component (F_y) of the cardiac activity force (CAF) of humans reported in an experimental paper by Silvia Conforto et al. [9]. This application of human cardiac data is simply due to the lack of sheep data. The heart rate is assumed to be constant at 80 beats per minute and the heart is assumed to arbitrarily stop at 45 s (7 s after the last breath) into the transient. The breathing frequency is arbitrarily assumed to be constant at a crisis rate of 30 breaths per minute and it is assumed to stop at 38 s based on Hollander's Figure 2. The external disturbance caused by breathing is expressed by $A_o \times \cos(\omega_b t)$, where ω_b is the angular frequency of breathing with amplitude A_o (= 0.345 N); this gives a vibration amplitude similar to that of the experiment. The disturbances assumed in the calculation at the last breath and after the stoppage of breathing will be explained in the calculated results.

Figure 3a shows the superposed external disturbance ($F_o \times f[t]$) assumed in the simulation with no disturbance after the square impulse. The high frequency disturbance caused by cardiac activity from the start to 38 s is modulated by the



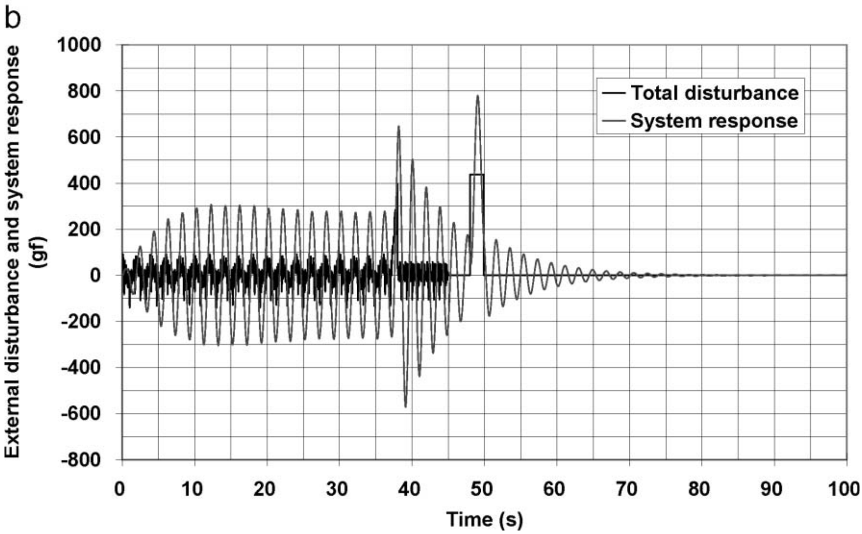


Fig. 3a. All external disturbances superposed in the simulation of the case with Sheep #7.
Fig. 3b. Simulation of case with Sheep #7.

lower frequency disturbance ($A_o \times \cos[\omega_b t]$) caused by breathing. The triangular impulse from 37 to 38 s reaches a maximum value of 4.04 N at 38 s (also adjusted to give a vibration amplitude similar to that of the experiment); the cardiac disturbance is superposed on the triangular impulse. The bottom width (1 s) of the triangular impulse is specified based on the corresponding experimental sharp pulse recorded around the time of the last breath. The disturbance shown from 38 to 45 s corresponds to the cardiac disturbance (F_y) only, and it shows a peak-to-peak swing of about 165 gf. The last square impulse has a width of $2\pi/\omega_n$ (=1.9 s), as defined in Sec. 2.2, and its height is adjusted to 4.3 N to give a peak pulse height of 780 gf.

The calculated system response is shown in Figure 3b, which also shows the superposed disturbance. The system response from $t = 0$ to $t = 37$ s (just before the triangular impulse) is determined primarily by the breathing disturbance.

The effect of cardiac disturbance is minimal because of the large ratio of the angular frequency of the cardiac disturbance (ω_c: more than 8.4 rad/s) to the natural frequency of the system ($\omega_n = 3.31$ rad/s). (About 6% of the 165 gf swing will be transmitted to the system response, if only F_y disturbance is considered in the simulation.) The theory of "vibration isolation" explains these behaviors in terms of the "amplification factor for transmissibility," $H_b(\omega_{ext}/\omega_n, \zeta)$, of vibration, where ω_{ext}/ω_n and ζ are the ratio of angular frequency of the external disturbance force to the natural one and the viscous damping factor of the system, respectively (see Dimarogonas & Haddad [7]). H_b implies that the amplitude of the external disturbance vibration (F_o) will be multiplied by the factor H_b in the output amplitude of the system response. The frequency ratio for the breathing disturbance, ω_b/ω_n, becomes 0.95, and that for cardiac disturbance, ω_{ext}/ω_n, becomes more than 2.5 because of the higher frequency components included in one cardiac cycle (according to Fourier analysis of F_y, $2\omega_c$, $3\omega_c$, $4\omega_c$, and $5\omega_c$ have larger weights than the fundamental harmonic ω_c calculated from the heart rate). In breathing disturbance, H_b becomes 7.8 for $\omega_b/\omega_n = 0.95$ with $\zeta = 0.042$, while in cardiac disturbance, $H_b = 0.19$ for $\omega_c/\omega_n = 2.5$. For a higher harmonic component of the cardiac disturbance, for example, $3\omega_c/\omega_n = 3 \times 2.5 = 7.6$, H_b becomes only 0.02. Hence, H_b roughly indicates the calculated results of the disturbance effects on the system response.

In Hollander (Figure 2), we can identify a higher frequency behavior during the time from about 19 to 28 s; the frequency is about two times the natural frequency of the system ($2 \times \omega_n = 2 \times 3.31$ rad/s). This means that there was a disturbance of corresponding frequency, to which the weighing system strongly responded, and this behavior may suggest that the weighing system has another component of natural frequency of about $2 \times \omega_n$ (which comes much closer to the cardiac frequency for 80 beats per minute than ω_n). This can be expected because the weighing system is composed of a two-dimensional plate and frame. However, this behavior cannot be simulated by the simplified single-degree-of-freedom model. When compared with the experimental result presented in Hollander's Figure 2, the present calculated result (Figure 3b) shows quite a different damped vibration after the triangular impulse at the last breath. The calculated damped vibration after the last square impulse at about 50 s into the transient appears similar to the experimental result. However, this is simply because of the artificially specified width of the impulse ($\Delta T = 2\pi/\omega_n$), as discussed in Sec. 2.2. If the width is specified wider or narrower than this value, damped vibrations similar to those calculated after the last breath will appear. Thus, this computer model can simulate only some aspects of the experiment.

(3) Possible cause of last impulse. The time integral of the last square impulse, $\int F(t)dt$, used in the calculation above gives a momentum of 8.16 N·s. However, half the ΔT with the same impulse height, that is, an impulse of 4.08 N·s is sufficient to give the 780 gf peak pulse, though the damped vibration after the impulse will be different. This can be seen from the analytical solution to the square impulse problem with no damping. If a free-fall event is assumed within the body of Sheep #7 after its apparent death to give this impulse, what would be the requirement with respect to a free fall?

Suppose that a mass "m" starts to fall freely through a vertical distance "h" in the gravitational field, and it stops delivering impact within the body when the distance is reached. The momentum of the mass at impact can be calculated as follows:

$$m \times v = m \times gt = m \times (2gh)^{0.5},$$

where v and g are the velocity at the impact and the gravitational constant (9.8 m/s^2), respectively.

To give an impulse of 4.08 N·s, the mass "m" and falling height "h" must satisfy the following relationship:

$$m \times v = m \times (2gh)^{0.5} = 4.08. \tag{3}$$

If the distance is specified as h = 5 cm, the required minimum mass becomes m = 4.1 kg for an impulse of 4.08 N·s. This result may not be consistent with Hollander's statement in his "Discussion" that "[experimentally] this requires a movement of several liters of fluid flowing relatively unobstructed to achieve a 50 to 100 gram transient pulse." This inconsistency between the two arguments may be because of the assumed width of the impulse (ΔT) being π/ω_n, which gives a 780-gf peak pulse with a minimum impulse $\int F(t)dt$ in the present speculation.

Could such a force event or its equivalent occur in the body of dead sheep? Could the ruminant system of sheep, which may contain fluidized food and possible gas accumulated in the process of fermentation of food, be responsible for such an event? Although this is just a speculation, we must seek out probable causes before concluding the experimental results as being "unexplained."

2.3.2. Simulation of Other Cases?

Hollander's paper shows the system responses for the cases with Sheep #3 and #8. Simulations of these cases with the present analytical model will provide no conclusions because the experimental results shown in his Figures 1 and 3 are quite different from the simulation result for Sheep #7. The weight of the experimental subject affects the simulated system response through the change in the natural frequency of the system (ω_n) and the term "F(t)/m" with different masses (m_{sp}) of the experimental subjects (see Equations 1 and 2). The ω_n's for Sheep #7 (m_{sp} = 70.2 kg), #3 (88.9 kg), and #8 (92.2 kg) are calculated to be 3.31, 3.12, and 3.09 rad/s, respectively, but the present model, with these effects of m_{sp} included, cannot give the peculiar vibration behaviors recorded in the cases with Sheep #3 and #8. Figures 1 and 3, if they experimentally present no problems, cannot be expected to be replicated by this simple theoretical model.

According to Hollander (Figure 1), the result with Sheep #3 is "a typical example of a transient occurring after the last deep breath and during a period of calm, free of any movement"; the result with Sheep #7 is rather exceptional. Regardless of the causes of the force impulses appearing in the experimental transients, that is, whether they are scientifically explainable or "unexplainable," we should suppose that the system responds to the impulses based on physical laws. Because the apparent responses of the system are very much peculiar in the cases with Sheep #3 and #8, there will be doubts about whether the weighing system functioned normally in those cases, including the cases with Sheep #3 and #8[1]. Normal functioning of the weighing system can be confirmed by conducting a system response test between successive cases by imparting a test disturbance to the system using an inert mass on the platform. Hollander did not mention such a test in his paper.

Although Hollander concluded that "there was no permanent weight change at death" in every case of the experiment, no reasoning for the conclusion was given. Definitely, the reported large escape rate of moisture from the subject bodies obscured any small anomalous change, if any, of the weight of the subjects upon death.

3. Simulation of Supposed Weight Measurement Experiment during OBEs

MacDougall's missing weights ranged from 10.6 to 70.8 (or 45.8) g [1][2], and they have neither been refuted nor proved in the last 100 years. This may support the assumption that there is a psycho-physical interaction between the "non-physical human mind" and the "physical body," and that part of the energy that accompanies the psycho-physical interaction and manifests in the physical dimension may be weighable in our gravitational field as a mass, ΔM, through Einstein's equation, $\Delta M = \Delta E/c^2$, where ΔE is the energy manifesting in the physical dimension. It should be noted here that when people talk about MacDougall's missing weights, they refer only to "the missing weight in the few seconds," which ranges from 10.6 to 42.5 (or 21.3) g [1], neglecting the additional decreases in weight that ensued in up to 18 minutes since the time of their judgment of death. (Hollander did so in his paper [6], for example.) The very additional missing weights are one of the "notorious points" against which the psychologist Blackmore stated her skepticism [2]. If one doubts the additional missing weights, the person should doubt the one observed in the few seconds too. However, unless there is a definite refutation of MacDougall's original results based on a scientifically quantitative basis, his experimental results should be respected. This is also because we do not know as yet the real meaning of "human death," when we take into account research results on "human reincarnation," for example, that by the late Prof. Ian Stevenson (1918–2007) [10].

MacDougall's experimental results may encourage weight measurement experiments in transitions to and from altered states of consciousness to show that in the transitions, there may be a violation of the Law of Conservation of Energy, which has been one of the most cherished *empirical principles* of physics.

3.1. Conditions for Simulation

Weight measurement experiments using a system like the one used by Hollander, for a physical body of human that is supposed to be left behind during OBEs, will be affected by disturbances caused by cardiac activity and body sway accompanied with breathing. The objective of the simulation is to clarify technical difficulties, if any, in such experiments. To make the simulation simple, it is assumed that the OBE experient is lying supine on the platform of the weighing system to minimize possible body sway. Only disturbances caused by the cardiac activity and breathing of the experient are taken into account. Based on the results of psycho-physiological research on OBE experients, the heart and breathing rates during the supposed OBE are assumed to be normal rates expected in the state of relaxation, although there are exceptional cases [11]. The normal weight of the OBE experient (m_{sp}) is assumed to be 62.0 kg.

(1) Cardiac disturbance. According to the science handbook [8], the heart rate of a human adult at rest ranges from 64 to 70 beats per minute. Based on this, the lower value of the range is selected in the simulation: 64 beats per minute for the heart rate.

The cardiac disturbance in the OBE experient *lying supine* may be expressed by the F_x component of the CAF published by Conforto et al. [9]. This component is the "frontward-backward" cardiac force obtained from experimental subjects standing upright. However, the most predominant CAF is the vertical F_y ("upward-downward") component in [9]; this component showed a peak-to-peak swing in the range of 1.3 to 3.0 N, depending on the experimental subjects. To maximize the possible cardiac disturbance in the simulation, F_y component with a peak-to-peak swing of 3.0 N is used (the F_y component used in Sec. 2.3 for the case with Sheep #7 is multiplied by 1.855 to get a peak-to-peak swing of 3.0 N). The time history of F_y in [9] is expressed as a function of the percent cardiac cycle. In the present simulation, F_y is given in the form of a table. The period of one cardiac cycle will be determined from the heart rate.

(2) Breathing disturbance. In the previous simulation of the case with Sheep #7, it has been shown that the breathing disturbance is dominant in the response of the weighing system. The situation will be different in the case of a human

OBE in a relaxed state, but as will be shown later, the disturbance caused by breathing will remain dominant in the system response.

Because there is no experimental data for breathing disturbance, a simple model is prepared for the following simulation study. The model, which is described in the Appendix, is intended to describe the up-down motion of the abdomen of a male subject in relation with the change in the air volume in the lungs during one breathing cycle. This up-down motion causes a cyclic force-impulse, $\int F(t)dt$, to which the weighing system may respond. The model assumes F(t) to be square-shaped with height F_1 (>0) and time width ΔT. The impulse $-F_1 \times \Delta T$ (upward) and $+F_1 \times \Delta T$ (downward) will be imposed on the system at the start of inhalation and at the turnaround to exhalation, respectively, in every breathing cycle. The time interval (TB_1) of inhalation is assumed to be one-third of a breathing period. The magnitude of $F_1 \times \Delta T$ has been evaluated in Table A1 of the Appendix. The evaluation shows that the impulse ranges from 0.034 to 0.101 N·s, depending primarily on the inhaled/exhaled air volume per breath, period of breathing, and the body mass participating in the up-down motion. The breathing rate of a human adult at rest ranges from 10.1 to 13.1 breaths per minute [8].

The height of the square impulse F_1 depends on the duration ΔT, which may range from 0.1 to 0.5 s according to muscle dynamics. However, calculations with changing ΔT have confirmed that the effect of breathing disturbance on the system response will be determined primarily by the magnitude of the impulse, $|F_1 \times \Delta T|$. Incidentally, the (adjusted) breathing disturbance at crisis ($F_{ext}[t] = A_o \times \cos[\omega_b t]$) used in the case with Sheep #7 in Sec. 2.3 corresponds to an alternating impulse of $\pm 2A_o/\omega_b = \pm 0.22$ N·s, which is about two times the maximum range of the impulse calculated by the simple model for a male human subject at rest. In the simulation, $|F_1 \times \Delta T| = 0.101$ N·s is used with a constant breathing rate of 13.1 breaths per minute to maximize the breathing disturbance.

Although these disturbance conditions may be inconsistent with the physiological state of an OBE experient in relaxation, these are assumed simply for the objective of this simulation to clarify technical difficulties caused by the disturbances in the supposed weight measurement experiment in OBEs. No other disturbance is assumed in the simulation.

As regards the supposed weight loss of the experient during OBE, it is assumed that a weight loss of 21 g due to OBE onset occurs instantaneously at $t = 100$ s and the weight returns at the termination of OBE at $t = 500$ s into the transient[3]; the calculation will be terminated at 600 s. In the simulation, a constant rate of decrease in the body weight due to insensible perspiration (i.e., moisture evaporating from the body during respiration and sweating) is assumed to be 31.5 g/h, which, based on standard physiological data, corresponds to 21% of the total daily heat loss (2100 kcal/day in a person in their 60s) from the body[4]. The time step size and convergence criterion used for the numerical solutions are similar to those used in Sec. 2.3.

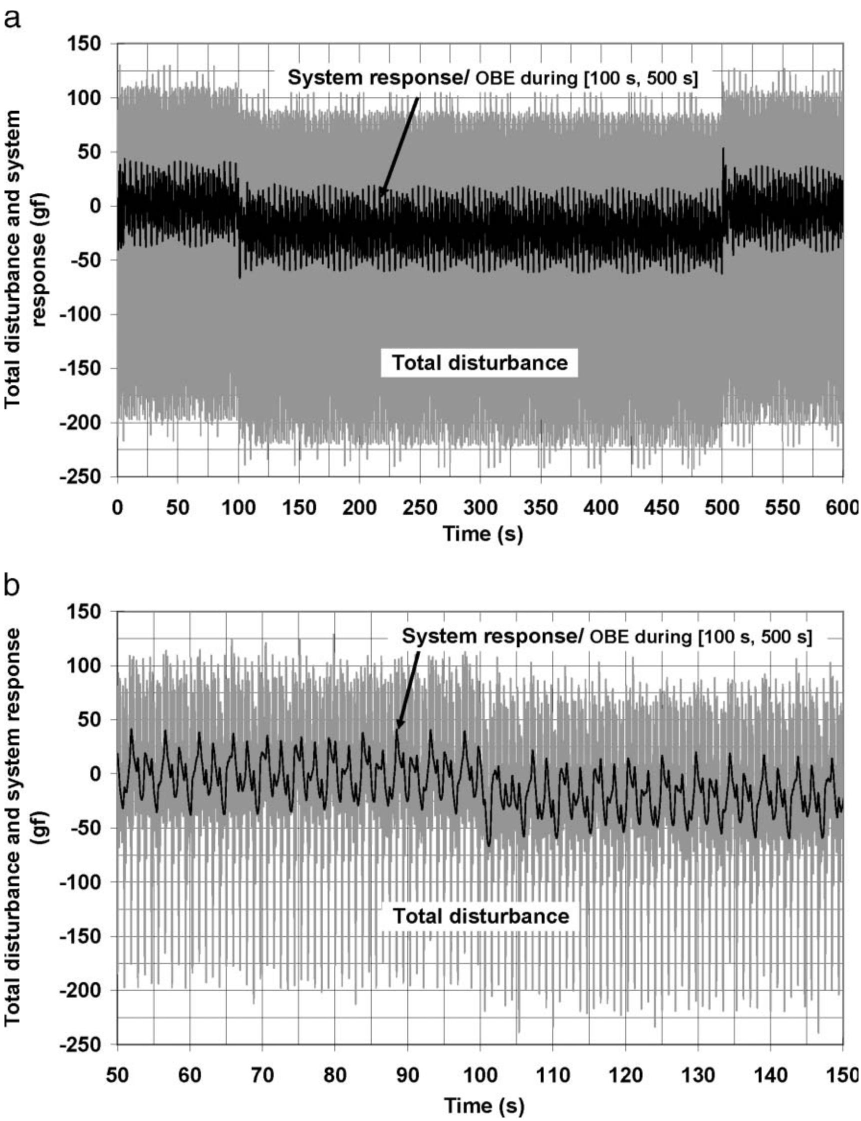


Fig. 4a. Simulation of supposed change in weight of OBE experient with weight decline at 31.5 g/h caused by insensible perspiration.
Fig. 4b. Part of the result shown in Figure 4a in expanded time scale.

3.2. Result of Simulation

(1) System response under disturbances. Figure 4a and b (4b is part of 4a in an expanded time scale) shows the result of simulation under the assumed disturbances. The calculated total disturbance showed a peak-to-peak swing of about

346 gf (3.39 N), out of which 306 gf (3 N) was caused by the high frequency cardiac disturbance, while the system response showed vibrations with a peak-to-peak swing of about 75 gf when the effect of damped vibration ceased. About 22% of the total disturbance is transmitted to the system response (if only the cardiac disturbance is assumed, about 8% will be transmitted). These vibrations in the system response are considered to be noise signals from the view point of the supposed experimental purpose. As seen in Figure 4a and b, the system response barely shows about 20 gf of decrease in the weight of the OBE experient under the effect of the maximized disturbances.

(2) Elimination of noise from calculated system response. So long as the disturbances are as simple as those assumed here, these noises can be easily eliminated from the system response to reveal the supposed change in weight of the OBE experient by applying some noise reduction techniques to the signals of the system response. To eliminate the noises caused by the disturbances from the system response (R[t]), an averaging method can be applied as follows:

$$\text{Averaged response: } < R(t') > \; = \; (1/TB) \times \int R(t)dt, \tag{4}$$

where the definite integral is calculated over one successive breathing cycle (t, t + TB), and time t′ is defined at the mid-point of the cycle interval. The period of breathing cycle (TB) is 4.58 s.

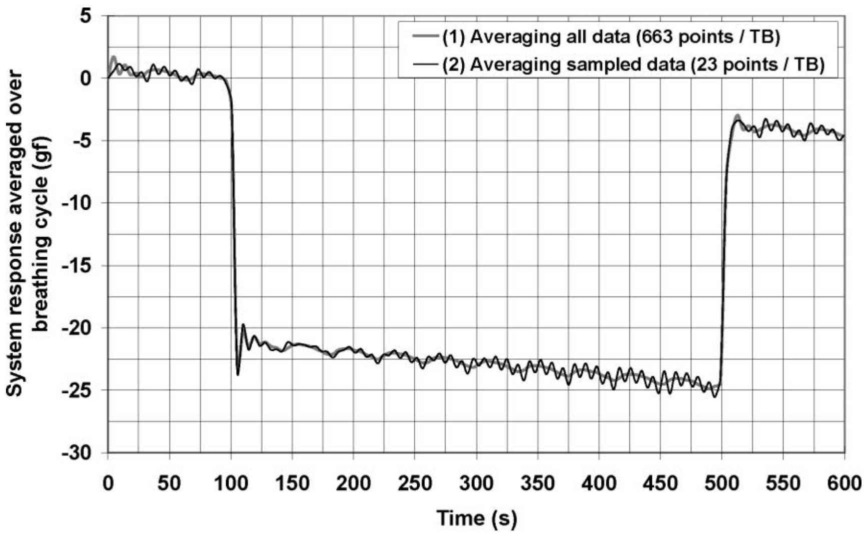


Fig. 5. Elimination of noise from the system response by averaging over each breathing cycle (TB = 4.58 s).

It should be noted that the period of cardiac cycle is about 0.94 s in this simulation; hence, the one breathing cycle covers almost five cardiac cycles. The system response expressed based on Equation 4 is shown in Figure 5 (1), which clearly shows the supposed change in weight of the OBE experient. Also shown in Figure 5 (2) is a similar average of the data points sampled from all calculated data at the rate of 5 points/s. This sampling is made based on the 0.2 s response time of Hollander's system. A small difference is seen between these two averaged results. However, these results show that if there really is some abrupt decrease in weight of the OBE experient of the order of tens of grams during OBE, the history of weight decrease can be discerned from the record of system response.

In actual experiments, we must also cope with the electrical drifting of instruments of the weighing system over the extended hours for which the experiments run; however, this issue can be dealt with technically.

However, no experimental result has been reported to show such a weight change of experients in any kind of transitions to and from altered states of consciousness, though only a very preliminary experimental report by John Hasted et al. [12] is available in this field of research. Why is this the case? Vernon Neppe and John Palmer contributed to a recently (in 2005) published book *Parapsychology in the Twenty-First Century* [13], writing an extensive review (up to the year 2002) and outlining future perspectives of research in the field of OBEs and near death experiences. However, they did not mention the possibility of an "*objective*" change in the weight of experients during such "*subjective*" paranormal experiences (SPEs in Neppe's term). Perhaps the lack of discussion on the subject explains the reason for only few reports being available.

An OBE as an SPE, however, is very much a *subjective* experience as compared to the SPE of "trance channeling," which can be obviously witnessed and controlled by experimenters during trance channeling sessions. This suggests that weight measurement experiments using a trance channeler, in comparison to OBE experiments, may be an easier way to obtain possible objective evidence, regardless of an increase or decrease in weight, of the violation of the Law of Conservation of Energy. However, the disturbances imparted to the weighing system will be greater in the former experiments.

Repetitions of MacDougall's experiments would be ethically forbidden today, and perhaps Hollander's type of experiments using animals will add no new value to this field of research, as shown by MacDougall, Twining, and even Hollander himself (since no anomalous change in weight upon death is not an exciting scientific result). Further, without independent confirmations of the "*missing weights*" of Duncan MacDougall, his experimental results mean almost nothing scientifically; this is the way of science. The only possibility of repeating MacDougall's type of experiments may be the use of weight measurement experiments in transitions to and from altered states of consciousness. Such repeatable experiments will be much more meaningful than experiments to "weigh the soul."

Authentic violation of the Law of Conservation of Energy in transitions to and from altered state of consciousness, if demonstrated, will provide a breakthrough in psychology as well as parapsychology. It will also compel scientists to investigate a new energy concept that can be used to understand both *psychic energy* and *physical energy,* fulfilling the dream of psychologist Carl G. Jung (1875–1961) [14].

4. Concluding Remarks

Based on parametric simulations of the case with Sheep #7 in Hollander's experiment, the following conclusions were drawn by using a simple analytical model of vibration for the experimental system:

(a) The experimental result with Sheep #7 appears very natural because the primary aspects of the result can be simulated theoretically.
(b) Hollander's conclusion that "there was a transient gain of weight of 780 grams" in the case of Sheep #7 is not an appropriate expression of the experimental result because the "780 gf pulse" includes an overshoot reaction of the weighing system. However, the cause of the force event remains to be explained. It was speculated that the force event might be explained based on a sporadic event possibly expected in the complex ruminant system of sheep even after death.
(c) The experimental results with Sheep #3 and #8 appear very strange from the viewpoint of theoretical prediction. It is doubtful whether the weighing system (primarily the four load cells) functioned normally. This question could have been answered if a system response test were conducted between successive cases with a test disturbance provided externally using an inert mass on the weighing platform[5].

Using the computer model for Hollander's weighing system, an analytical simulation of a supposed weight measurement experiment was conducted for an OBE subject, assuming a weight loss of 21 g during OBE. The simulation showed that the disturbance probably caused by breathing becomes the primary noise rather than the noise from cardiac disturbance affecting the system response. However, it was shown that some noise reduction techniques can discern the change in weight of an OBE experient, if there really is a weight decrease of tens of grams during OBEs. The present author would like to suggest weight measurement experiments using a trance channeler, because trance channeling is objectively more observable a phenomenon than OBEs.

Notes

[1] The frequency of data acquisition of the experimental system was 2 Hz for Sheep #3; it was 21 Hz for Sheep #7 and #8. It is confirmed, however, that even if the calculated system response shown in Figure 3b is expressed with data points sampled at the rate of 2 points (out of 133) per second, the apparent result is not changed much.

[2] The maximum range of 70.8 g originated from MacDougall's third patient. However, there is an ambiguity with regard to the language MacDougall used to describe the third patient: MacDougall wrote "My third case, a man dying of tuberculosis, showed a weight of half and ounce lost, coincident with death, and an additional loss of 1 ounce a few minutes later." [Underline added.] The ambiguity lies in the expression "half and ounce," which should have read "half and an ounce" if the loss was 1.5 oz. Thus, if the correct expression is "half an ounce," then the maximum range would originate from the second patient (45.8 g).

[3] The 21 g is 0.034% of 62.0 kg, and this fraction is well over the scale sensitivity of Hollander's system (−5 g with the full-scale capacity of 100 kg).

[4] Vapor mass loss rate, M′, corresponding to this heat loss rate (Q_{ip} = 441 kcal/day) can be calculated as $M' = Q_{ip}/\Delta_{VAP}H = 31.5$ g/h, where $\Delta_{VAP}H$ is the latent heat of water vaporization (=2.444 kJ/g = 0.584 kcal/g at 25°C).

[5] Hollander might have conducted such a test. However, he did not mention it in his paper [6].

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APPENDIX

A Simple Model for the Breathing Disturbance

It is known that the surface boundary of the abdomen of a male human shows cyclic up and down motion during the breathing cycle. The model is intended to describe this up-down motion in relation to the change in the gas volume in lungs during one breathing cycle. Let us define the initial state of the gas volume as the exhaled state of the lungs and look at the change in the molar gas volume (air + CO_2), which changes as the moles of gas change. For simplicity, the rates of air inhalation and gas exhalation are assumed to be constant during inhalation and exhalation, respectively. Inhalation continues from t (time) = 0 to t = TB_1 and exhalation immediately follows until t = $TB_1 + TB_2 = TB$, which is the period of one breathing cycle. TB_1/TB may be (1/3) in a relaxed state. The gas volume V_b will be given as follows from the ideal gas law:

$$V_b = n \times RT/P_a, \tag{A1}$$

where

P_a = pressure of the gas in the volume; assumed to be atmospheric;
V_b = the gas volume (not including the dead gas volume of the lungs);
n = moles of gas in the volume;
R = ideal gas constant;
T = gas temperature in K.

Air intake and gas discharge are facilitated by the change in the negative pressure in the pleural cavity caused by the contraction/expansion of the diaphragm and intercostal muscles. However, we do not need to get into the details of the breathing mechanism for this simple model. The up-down motion of a part of the body mass in the abdomen may cause a dynamic disturbance, to which the weighing system responds.

The model assumes that the body of the experimental subject is lying supine on the platform of the weighing system, and that the body mass M_b participating in the motion is the horizontal upper half of the abdomen with a horizontal length L_o. Approximating the shape of the cross section of the abdomen to be an ellipse with a semi-major axis "a" and semi-minor axis "b," mass M_b may be expressed as

$$M_b = (1/2) \times \pi ab \times L_o \times \langle\rho\rangle, \tag{A2}$$

where $\langle\rho\rangle$ is the average density of the body.

The model assumes that the center of mass of M_b will be displaced vertically by a distance δx due to an increase in the gas volume δV_b as follows:

$$\delta x \times S_o = \delta V_b,$$

where $S_o = L_o \times 2a$ and δx is defined as positive for vertically upward displacement.

This relationship leads to the following differential equation:

$$\delta V_b/\delta t = S_o \times \delta x/\delta t, \tag{A3}$$

where δt is the time interval in which δV_b occurs.

The left side of Equation A3 is proportional to air intake rate, (dn/dt), during inhalation, and to gas discharge rate during exhalation.

The momentum (P) of the mass M_b may be expressed by using Equations A1, A2, and A3 as follows:

$$\begin{aligned} P &= M_b \times \delta x/\delta t = M_b \times [\delta V_b/\delta t]/S_o \\ &= (1/2) \times \pi ab \times L_o \times <\rho> /S_o \times [RT/P_a] \times dn/dt \qquad (A4) \\ &= (1/4) \times \pi b \times <\rho> \times [RT/P_a] \times dn/dt \end{aligned}$$

Because the gas intake and discharge rates, dn/dt, are assumed to be constant during both inhalation and exhalation, the momentum of the mass will be constant during both inhalation and exhalation; the former momentum is expressed as P_1, and the latter as P_2. This change in momentum will be repeated cyclically with the period of breathing, TB.

V_b will reach its maximum, $V_{b\,max}$, at the end of inhalation with the maximum number of moles of gas, n_{max}. Then, the assumed constant rate, dn/dt, will be expressed as follows:

$$\begin{aligned} dn/dt &= n_{max}/TB_1 = V_{b\,max} \times [P_a/RT]/TB_1 \text{ during inhalation,} \\ &= -n_{max}/TB_2 = -V_{b\,max} \times [P_a/RT]/TB_2 \text{ during exhalation.} \end{aligned}$$

$V_{b\,max}$ is the air volume breathed per breath. Then, the momenta P_1 and P_2 will be expressed as follows:

$$\begin{aligned} P_1 &= +(1/4) \times \pi b \times <\rho> \times V_{b\,max}/TB_1 \\ P_2 &= -(1/4) \times \pi b \times <\rho> \times V_{b\,max}/TB_2 \end{aligned}$$

The cyclic momentum of the mass cannot change from P_2 to P_1 at the start of inhalation; neither can it change from P_1 to P_2 at the turnaround to exhalation without some action. There must be an impulse force, F(t), acting to cause the change in momentum:

$$\Delta P_{in} \equiv (P_1 - P_2) = \int F_{in}(t)dt \text{ at the start of inhalation,}$$

$$\Delta P_{ex} \equiv (P_2 - P_1) = \int F_{ex}(t)dt \text{ at the turnaround to exhalation.}$$

These impulses may be caused by the actions of some muscles relevant to breathing. Assuming square-shaped impulses of $F_{in}(t)$ and $F_{ex}(t)$ with the same time duration ΔT for model simplicity, the height of the impulse will be expressed as follows:

$$F_{in}(t) = F_1 \text{ during } \Delta T \text{ at the start of inhalation,}$$
$$F_{ex}(t) = -F_1 \text{ during } \Delta T \text{ at the turnaround to exhalation.}$$

F_1 will be expressed as follows using the expressions for P_1 and P_2 given above:

$$\begin{aligned} F_1 \times \Delta T &= (P_1 - P_2) \\ &= (1/4) \times \pi b \times \langle \rho \rangle \times V_{b\,max} / TB / [x(1-x)], \end{aligned} \qquad \text{(A5a)}$$

where $x \equiv TB_1/TB$.

If $TB_1 = TB_2$, the impulse $F_1 \times \Delta T$ is expressed as

$$F_1 \times \Delta T = \pi b \times \langle \rho \rangle \times V_{b\,max} / TB. \qquad \text{(A5b)}$$

The following data have to be specified as input to use in Equation A5a:

$V_{b\,max}$ = gas volume inhaled/exhaled per one breathing cycle (m^3);
TB = length of one breathing cycle (s);
$x \equiv TB_1/TB$ = fraction of inhalation time interval per breathing cycle; $x = (1/3)$ will be used for a typical adult at rest (note that function $1/[x(1-x)]$ has a very flat bottom in the range of $x = 0.3$ to 0.7 with the minimum 0.4 at $x = 0.5$);
$\langle \rho \rangle$ = average density of human body; assumed to be equal to the density of 4% saltwater density at 20°C: 1027 kg/m^3;
b = length of minor axis of the elliptic cross section of the abdomen (m);
ΔT = duration (s) of the square impulse F_1; will be treated as a parameter.

Table A1 shows basic biological data used to obtain TB and $V_{b\,max}$ for the evaluation of the required impulse, $F_1 \times \Delta T$. (The sheep data in the top table (A) are shown for comparison.) The estimated ranges of impulse, $F_1 \times \Delta T$, are shown in the bottom table (B) for the cases of human male subjects, who may show greater impulses than female subjects. The impulses range from 0.034 to 0.101 N·s. The height of the square impulse F_1 depends upon the duration ΔT, which may range from 0.1 to 0.5 s based on muscle dynamics. However, it can be shown that the effect of the breathing disturbance on the system response primarily depends on the magnitude of the impulse, $|F_1 \times \Delta T|$.

TABLE A1

Basic Biological Data and Results of the Impulse Model for the Breathing Disturbance

(A) Biological Data of Breathing of Humans and Sheep at Rest (Based on Data in a Science Handbook [8])

	Human male (at rest)		Human female (at rest)		Sheep (at rest)	
Item	Min	Max	Min	Max	Min	Max
Air volume breathed per minute (liters/minute)	5.8	10.3	4	5.1	5.95	7.69
Breaths per minute (1/minute)	10.1	13.1	10.4	13	15.7	23.6
TB (s)	4.58	5.94	4.62	5.77	2.54	3.82
$V_{b\ max}$ (liters/breath)	0.443	1.020	0.308	0.490	0.252	0.490
Air mass per breath (g/breath)	0.505	1.164	0.351	0.560	0.288	0.559
Average TB (s)	5.172		5.128		3.053	
Average $V_{b\ max}$ (liters/breath)	0.694		0.389		0.347	

Note: Only the first two rows are from [8]; the min and max simply correspond to the ranges obtained from [8]. The data in the third row and below are calculated based on the first two rows' data; min, max, and average show only the possible ranges and average calculated using the min and max data in the first two rows. Atmospheric air density = 1.1415 kg/m^3 at 37°C. TB = period of breathing cycle; $V_{b\ max}$ = air volume breathed per breath.

(B) Evaluation of the Impulse Using Human Male Data

Data	Case 1	Case 2	Case 3
$V_{b\ max}$ (m^3/breath)[a]	6.940E−04	1.020E−03	4.430E−04
TB (s)	5.17	4.58	5.94
$x \equiv TB_1/TB$ (−)	1/3	1/3	1/3
$<\rho>$ (kg/m^3)	1027	1027	1027
b (m)[b]	0.125	0.125	0.125
Impulse $F_1 \times \Delta T$ (N·s)[c]	0.061	0.101	0.034
Note	With average data pair of TB & $V_{b\ max}$	With pair to give max impulse	With pair to give min impulse

Note: $V_{b\ max}$ = air volume breathed per breath; TB_1 = time interval of inhalation; TB = period of breathing cycle; $<\rho>$ = average density of body; b = half abdomen thickness; F_1 = height of impulse; ΔT = duration of impulse.

[a] "6.940E−04," for example, stands for 6.940×10^{-4}.

[b] Half abdomen thickness "b" = 0.125 m is based on the present author's body.

[c] The impulse by the model for the body lying supine on the platform: $F_1 \times \Delta T = (1/4) \times \pi b \times <\rho> \times V_{b\ max}/TB/[x(1-x)]$, where $x = TB_1/TB$. The duration of impulse ΔT, possibly ranging from 0.1 to 0.5 s, does not much affect the simulation.