Mathematics: The Loss of Certainty by Morris Kline. New York: Fall River Press, 2011 [originally published in 1980 by Oxford University Press]. 464 pp. $19.95 (paperback). ISBN 9781435136069.

In 1980 Morris Kline wrote this engaging book, in which he took on many of the myths about the nature and history of mathematics. This new edition will probably be as seldom read as the original, which is too bad because it contains important messages, including perhaps some comfort for anomalies researchers. I will briefly present an overview of the book's contents, and then say what I think these comforts are.

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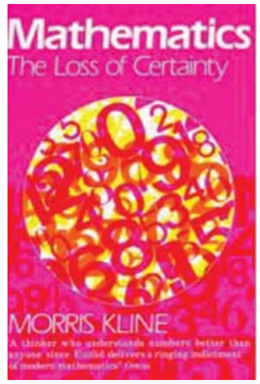
The ancient Greeks developed the seed of what we now think of as mathematics. Kline points out that their mathematical concepts arose from consideration of the natural world, and then the fact that numbers, shapes, and relationships corresponded to things in the real world convinced them that reality itself was in some mystical way generated by numerical principles. The regular patterns that they found in geometric forms and simple integers reflected the regularities of nature, and so provided keys to understanding how things were, and why they were that way. The faith that mathematics lay behind the mundane world of observations became an unquestioned truth, at least as important as the practical techniques the Greeks devised, and passed along through the Middle East and the medieval period to modern Europe.

Of course Euclid's *Elements* was the foundation of the Greek legacy. There is no question about the fact that it was designed in order to describe the spatial aspects of the world we live in. It was not a hodge-podge of facts bound loosely together because they all pertained to space, but rather an intricate structure, in which one started with definitions that clearly applied to real things (points, lines, and so on), and then through the power of deductive logic alone one discovered and even proved things that could be observed in the real world. As a model for what a deductive system should look like, it persevered well into the modern period in Europe. But perhaps more important than its specific insights and theorems, it justified a view of mathematics as an engine with which the human mind could understand the natural world. It was a short step from there to believe that the natural world was designed and created on the basis of Euclid's geometric truths. Since the religionists of modern Europe were quite eager to obtain a monopoly on the truth, they had little difficulty convincing themselves that the hand of the Creator was to be seen in this remarkable relationship between apparently abstract mathematics and concrete reality.



The idea that one learns about nature through observation and experiment developed slowly during the early modern period, but, as Kline argues, if mathematics represents truth, and truth is exemplified in scientific observations, then mathematics must be the appropriate language for talking about science. Therefore, one whole strain of the development of mathematics, from the 15th to the beginning of the 19th centuries involved the increasing mathematization of science. The success of this enterprise had the effect of bolstering the belief that mathematics and truth were necessarily bound together.

The first crack in this world view came in the 17th century, when a number of mathematicians simultaneously developed calculus (although Newton and Leibniz usually receive most of the credit). Kline does not mention it, but the basic ideas of calculus go back to Archimedes, who failed to invent it primarily due to an inadequate number system. All versions of calculus involved taking ratios of things where both the numerator and denominator tend to 0. The problem was in claiming that this operation had some kind of legitimacy. One might say that something like 1/0 could be interpreted as infinity (whatever that was), but 0/0 would not yield to any sensible interpretation. The overwhelming fact about calculus was, however, that it was immensely useful. From the logical standpoint, this was a muddle, since one seemed to be performing nonsensical steps to consistently obtain correct answers. Virtually all of the arguments about the nonsensical steps were metaphysical, both on the side of Newton, Leibniz, and their adherents, as well as on the side of the opponents, notably Bishop Berkeley. As Kline points out, this unsatisfactory situation continued quite persistently for at least two centuries, until Cauchy provided the modern definition of a "limit."

Despite the saving of calculus by Cauchy, Kline sees an even further unraveling of the logical status of mathematics in the 19th century. The first difficulty with the "mathematics = truth" equation was created by Hamilton in 1843 when he invented quaternions. He was trying to address exactly the same kinds of problems as Euclid, the description of three-dimensional space, but using algebraic methods rather than deductive geometry. Quaternions are intimately bound up with rotations, and as anyone familiar with Rubik's Cube has discovered, rotations in three dimensions are not commutative (the order in which you perform a sequence of rotations is important to the result). Mathematicians in Hamilton's time were so committed to the idea that arithmetic (as they had learned it) was truth, that it was illogical (if not blasphemous) to talk of multiplication being non-commutative. Despite the historical importance of quaternions, they tended to fall by the wayside, only to be rediscovered recently in applications to aircraft electronics and video games.

The second difficulty was closer to a disaster. The lore of centuries had held that Euclid's geometry was the one and only true geometry. But there were a few things that were not entirely clear. One was whether Euclid's axioms were independent of each other. Of particular concern was the "parallel postulate," which can be stated several different ways. This axiom seemed to many mathematicians to be less self-evident than Euclid's other assumptions, and since doubt appears as the enemy of truth, it was important to clear the matter up. One way of looking at the problem was to ask whether the parallel postulate could be deduced from the other axioms. If so, then it could be discarded as an axiom, and all the rest of Euclid's work would remain as it was. But if not, the possibility presented itself that one might be able to state Euclid's geometry in a form that simply did away with the parallel postulate, or perhaps replaced it with a different version. This latter step was taken by a number of 19th century mathematicians, creating a variety of "non-Euclidean" geometries. Again Kline points out that most mathematicians rejected these geometries as novelties, because they held to the "mathematics = truth" belief, and the real world was obviously Euclidean. When a special case of Riemann's elliptical geometry was seen to apply to the surface of a sphere (where "straight line" means "great circle"), then because the sphere was also a part of Euclid's geometry, the tide turned in favor of acceptance of non-Euclidean geometries.

Having seen their discipline pass successfully through several challenging storms, the mathematicians of the early 20th century expressed supreme confidence that all of the potential logical problems with mathematics had been dealt with, and all that remained (in the words of Lord Kelvin) was to fill in the details. Figures such as Bertrand Russell and David Hilbert undertook the task of putting all of mathematics on a solid foundation of some version of logic. Hilbert in particular was certain that his "proof theory," which we now call "formal systems," was the correct way forward, and so he proclaimed that the end of the period of uncertainties in mathematics was at hand.

But once again fate conspired to dash such noble hopes. In the early 1930s Kurt Gödel proved that any system at least as complicated as arithmetic (in the mathematical sense) was either inconsistent (one could deduce contradictions) or incomplete (there were true statements that could never be proved from within the system). For centuries one of the most troubling uncertainties that had bothered mathematicians was whether or not the systems of thought they had inherited were consistent. If Euclidean geometry was inconsistent, then it could not describe space (which evidently is consistent), but how could we tell? What was needed was a methodology for testing whether a given system was consistent. But Gödel's result then said that if you achieved this for some system, you would have simultaneously found that there were truths in the system that could never be discovered, within the system. Applying this to all of mathematics, Gödel had shown that Hilbert's program of reducing mathematics to formal systems was doomed. While on the one hand we can see Gödel's result as a triumph of mathematics, on the other hand the victory seemed remarkably Pyrrhic.

Kline finishes his narrative with observations on the foundations of mathematics, especially the "axiom of choice" and Cantor's "continuum hypothesis." His opinions come out most strongly in the later chapters where he rails against the modern tendency of mathematicians to value abstract, literally useless creativity, as opposed to the direction of mathematics back to its roots, the solution of actual problems.

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Why should any of this be of interest to anomalies researchers? I think that much of it is, for the simple reason that woven in among the major themes I've described above, Kline includes rich detail about just how confused most mathematicians have been throughout most of the history of their discipline. He points out how vague Euclid's actual definitions and axioms really are, and how much trouble this created for those who wanted to see geometry as being logically tight. He also emphasizes a fact that so far as I know has been completely omitted from mathematics texts and virtually all histories. Although negative numbers were known in medieval times, and the necessity of the square root of minus 1 since the 15th century, it was not until perhaps the late 18th century that mathematicians began to accept these as numbers. For the complex numbers, one can perhaps understand the reluctance, although Euler had already shown that they were no more complex than the two-dimensional plane. But the inability to conceive of the use of negatives shows a truly remarkable failure to employ the one feature of mathematics that everyone agreed to—that it should portray reality. It is even more astonishing to realize that Newton developed calculus while believing that subtraction of a larger number from a smaller one was a meaningless operation. Even further, insofar as mathematics being the model of a logical deductive system, all of the great mathematicians used complex numbers, and sometimes even negative numbers, to obtain their results. Again we see logic sacrificed for the sake of practicality, and the hypocrisy of claiming the infallibility of the results because they were supposed to arise from a logically pure source.

It does not stop there. Consider the fundamental point in the definition of calculus, that ratios of quantities each approaching zero gives an uninterpretable 0/0. Consider the ratio $(1 – x^2)/(1 – x)$. As x goes to 1, the numerator and denominator each go to 0. But the expression is equal to 1 + x, and everyone agreed that this goes to 1 as x goes to 0. In other words, there are pathetically simple examples to demonstrate that there is not necessarily any problem with the Newton/Liebniz infinitesimal ratios. Moreover, Newton believed that all continuous functions were differentiable, flying n the face of truly trivial realistic counterexamples. I have seen this same pattern come up in how scholars of this and subsequent eras dealt with questions about probability. Often they endlessly debated points with barrages of philosophical arguments, when a few simple examples would have made the situation abundantly clear.

Although Kline mentions the fact that even into the 19th century mathematicians were confused about discontinuous functions, he does not mention the famous story about Fourier. In his investigation of the propagation of heat, Fourier asserted that any function could be approximated by a series of sines and cosines. His assertion so offended the leading lights of his day that its publication was blocked for more than a decade. (He was almost right; the notion of pointwise convergence needs to be replaced by convergence in L_2 norm). Kline does, however, devote a section to how confused even the great mathematicians were about series (of numbers, not even functions).

Here is the lesson that I take from Kline's history. In mathematics we have an excellent example of a method of thinking that laid claim to absolute truth, while it was in fact often wallowing in confusion and error. The situation was complicated by the fact that much of the mathematics that was created was both subtle and incredibly useful. But this turned out to be a double-edged sword, since every advance brought with it further confidence in the underlying logic, and simply postponed the day of reckoning. The proponents of mathematics went vastly beyond the facts in their crowning of it as the "queen of sciences," and were thus largely blind and resistant to most of the attempts to remove the evident problems. The history of mathematics is not what is taught in elementary science classes, an inexorable march of progress, but instead it lurched from success to disaster to success . . . and so on for centuries.

As a final example of the hubris of conventional science, we can cite the topic in dynamic systems theory somewhat inappropriately called "chaos." This could not have been covered by Kline, because Edward Lorenz did not make his celebrated rediscovery of the phenomenon until several years after Kline's book was published (and Kline died in 1992). But it would have suited Kline's purpose admirably, since it was Henri Poincaré, just after the turn of the 19th century who discovered and fully appreciated the essential unsolvability of certain easily stated physical problems. Poincaré turned away from the abyss, and for 70 years no one else sneaked up to the edge to take a peek. And that, I think, helps to define the role of anomalists in the 21st century; they are the ones who go up to the edge and peek.

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