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Article of Interest

On the Reality of the Quantum State by M. F. Pusey, J. Barrett, and T. Rudolph, *Nature Physics*, *8*, 476–479, May 2012. doi:10.1038/nphys2309.

Is the Quantum Wave Function Real?

Reality? What a concept.

—Robin Williams

Introduction

The conflict between all we know about the physics of quantum systems and what we say or believe is real about them is brought forward dramatically with the concept of the quantum wave function (QWF). Is the QWF ontic or merely epistemic? Here I review and clarify through original examples the recent work of M. F. Pusey, J. Barrett, and T. Rudolph who have formed a novel theorem to decide on the ontology or epistemology of a QWF based on a hidden variable (HV) theory dating back to the mid-20th century.

In a remarkable remark, physicist E. T. Jaynes once stated:

> We believe that to achieve a rational picture of the world it is necessary to set up another clear division of labor within theoretical physics; it is the job of the laws of physics to describe physical causation at the level of ontology, and the job of probability theory to describe human inferences at the level of epistemology. The Copenhagen interpretation scrambles these very different functions into a nasty omelet in which the distinction between reality and our knowledge of reality is lost. (Jaynes 1989)

I shall use the adjectives *ontic* and *epistemic* to modify a number of nouns such as physics, beliefs, observables, and reality as most of us currently understand these things. Hence ontic reality is what we accept as real and "out there" objectively independent of anything we have to say, believe, or know about it. Epistemic reality, on the other hand, is what we accept as real and "in here" subjectively dependent on what we think, know, or believe is either ontic or epistemic reality.

Into this omelet we now add some new ingredients, or perhaps better said, we give the omelet another flip in the frying pan. Is the quantum wave function (QWF) epistemologically or ontologically real? In a recent *Nature* review, E. S. Reich (2012) discussed the latest work (the article under review here) of three physicists: M. F. Pusey, J. Barrett, and T. Rudolph

(PBR). PBR, basing their work on a number of previous epistemic vs. ontic considerations dating all the way back to the Einstein–Bohr debate at the 1927 Solvay conference in Brussels and continuing with the 20th- and 21st-century work of many others, notably Bell, Bohm, Caves, Fuchs, Harrigan and Spekkens, Kochen and Specker, Norsen, and others, once again throws down the gauntlet of uncertainty by attempting to provide an ontic view of the QWF, something that even Bohr most likely was not ever considering. Jaynes even pointed out that the famous Bohr–Einstein debate was actually never resolved in favor of Bohr at Solvay in 1927—although common thinking even among physicists is that it was—when you consider that the two physicists were not discussing the same physics. Bohr was only thinking about epistemic physics while Einstein was considering only ontic physics. Hence while Bohr believed quantum physics was certainly epistemically complete (like classical thermodynamics), Einstein was equally correct in believing that quantum physics wasn't ontologically complete (like Newtonian mechanics).

The conflict between all we know about the physics of quantum systems and what we say or believe is real about them is brought forward dramatically with the concept of the QWF. Is the QWF ontic or merely epistemic? To decide on the ontology or epistemology of a QWF, an old argument known as the hidden variable (HV) theory dating back to the mid-20th century is revisited. This theory was probably most emphasized by David Bohm (who formulated from standard quantum physics an ontic QWF that influenced a real particle). Later it was revisited by Bell, in his famous no-go theorem involving a QWF describing two quantum-entangled separated particles à la Bohm's version of the Einstein, Podolsky, and Rosen (BEPR) paradox. BEPR showed that such a QWF could not be local (measurements made on one particle at one spacetime location could influence and change the QWF and therefore the outcome of measurement on the other particle at a distant (spacelike) spacetime location simultaneously). Bell's theorem shows that any hidden variable theory must involve nonlocal influences at the ontic level, regardless of what you think of the QWF. Hence one might conclude from Bell's famous HV theorem (à la Einstein) that QWFs are epistemological rather than ontological since two observers could have different beliefs about the quantum state of their respective spacelike separated particles.[1]

Quantum physical HV theories all have one thing in common: They all have ontic definite-valued hidden states underlying the QWF. A specification of these HVs should reveal the results of a measurement of any property or observable.[2] So the question is what would one need to do to HV theory to make the QWF ontological? This is precisely what PBR do by making a particular assumption: If a specification of an HV uniquely determines a QWF, then the QWF is ontic. If, on the other hand, specification of an HV does not uniquely determine a QWF, the QWF is said to be epistemic.

As an epistemic example, in the first version of their paper (Pusey, Barrett, & Rudolph 2012), PBR consider a classical case of flipping a biased coin in one of two distinct ways. In the first way the coin has a probability p_1 of coming up heads while in the second way the probability for heads is $p_2 \neq p_1$. If the coin is flipped and then observed any number of times, regardless of the results obtained, we cannot know for certain by which method the coin was flipped, although the observed frequency of heads resulting could provide a clue, provided we knew that the same preparation was used each flip. Not knowing this, the result, heads, could have been obtained with either mode of flipping. Hence we cannot assign uniquely either probability p_2 or p_1 and these probabilities remain epistemic although the unobserved method of flipping need not be so.

In another epistemic example, used by Reich in her review (Reich 2012), consider a die prepared in a manner that shows the value 2 with a predicted probability of ⅓. We cannot know if the die was prepared in such a way that only prime numbers (2, 3, or 5) were allowed to show, or if only even numbers (2, 4, or 6) were allowed to show. Each distribution has the number 2 in common, so the distributions are conjoint and epistemic.

Classical Physics Epistemics

Let me now give you a simple example of the difference between ontic and epistemic reality taken from classical physics. Consider a ball with mass m = ½ attached to a spring with spring constant k = 2. Such a system is known as a simple harmonic oscillator (SHO)—stretch or compress the spring and the SHO "springs" into motion with the ball having momentum *p* and a position *x* relative to its unstretched or uncompressed 0 position, and constant energy $E = p^2 + x^2$. I'll use a single variable λ to denote the ontic pair (*p*, *x*). Suppose that someone unknown to us stretches the spring an unknown initial distance, x_0, within a range $1 \leq x_0 \leq 2$ or in a second range $3 \leq x_0 \leq 4$. If you think of a two-dimensional space with orthogonal coordinate axes, *p* and *x*, the above energy equation describes a circle contained within one of the two sets of concentric thickened circles centered about the coordinate origin. Such a space is a simple example of what is called a phase space which in general has n dimensions of *p*s and *x*s. Every point on a circle provides a momentum and position of the ball which, even if not observed, hence hidden, are ontic variables. At no time do the different sets of circles have common points of overlap.

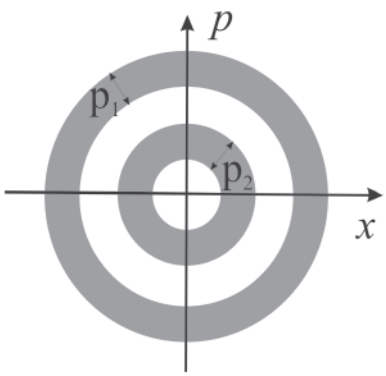

Figure 1. Disjoint epistemic probability distributions in phase space for a SHO (see text.)

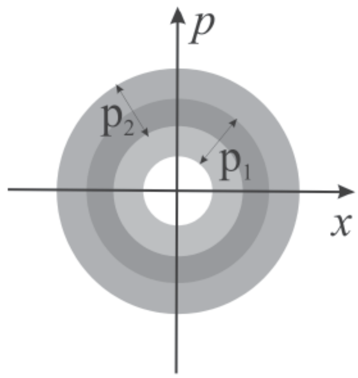

Figure 2. Conjoint (overlapping dark grey) epistemic probability distributions in phase space for a SHO (see text).

We can think of the thickened circles as disjoint probability distributions, $p_1(\lambda)$ and $p_2(\lambda)$, of positions and momenta—disjoint because we never have any λs in common—the thick circles are concentrically nested (see Figure 1). Each λ may be a uniformly distributed (over time) HV satisfying the SHO energy equation (and the Liouville equation in phase space if we didn't know the energies which govern the temporal evolution of more complex distributions involving more SHOs). However, as I said, these simple distributions would be disjointed. Hence $p_1(\lambda) \bullet p_2(\lambda) = 0$ always since each λ uniquely determines its own distribution. Consequently if there was a state α_1 associated with $p_1(\lambda)$ and a state α_2 associated with $p_2(\lambda)$, then specification of the value of λ would uniquely determine which state, α_1 or α_2, we would be in. We could, although it is clearly not necessary, view the λs as HVs and declare the states as ontic since each λ uniquely determines α.

Suppose we now reconsider the initial preparation of the SHO. At t = 0, that unknown someone simply decides to stretch the spring a certain distance, x_0, an amount in the range, $1 \leq x_0 \leq 3$, and lets it go.[3] We would then find a thick ring band of different energy possibilities in the phase plane. Or if the unknown person prepares the SHO in the range $2 \leq x_0 \leq 4$ and lets it go, we would then find a second thick ring band of possibilities. The two circular bands now form overlapping concentrically nested distributions (see Figure 2). Now we have the two distributions, $p_1(\lambda)$ and $p_2(\lambda)$, overlapping. Then $p_1(\lambda) \bullet p_2(\lambda) \neq 0$ in the overlapping area $2 \leq x_0 \leq 3$ and each λ no longer uniquely determines its own state. A specification of λ in the overlapping probability distribution could indicate we were in either the α_1 or α_2 state and that would make the states epistemic.

PBR's proof is based on a contradiction that arises between the probability predictions of quantum physics when QWFs are considered to be ontological (their respective HV probability distributions are disjoint) and the same predictions based on epistemic QWFs (their respective HV probability distributions are conjoint). They consider this contradiction in a series of ever increasingly complex arguments that includes a calculation eventually involving n identically prepared and uncorrelated independent states as well as noise considerations. Accordingly, whenever QWFs of observables are governed by disjoint distributions of ontic HVs, these QWFs are uniquely determined and must be ontic even though their respective distributions are epistemic (similar to arguments made in statistical mechanics). Thus if the states of a quantum system are specified by QWFs which are determined by disjoint epistemic distributions over ontic variables, the QWFs are as ontic or real as any observable in physics. On the other hand, if such distributions governing these QWFs are conjoint, that is they have values of ontic HVs in common, the QWFs are epistemic or merely represent knowledge (probabilities) of observables in question.

Simple Quantum Physics Ontology and Epistemology

Before we look at PBR's argument, I want to explain a little more about why overlapping probability distributions lead to a contradiction in the quantum physical predictions. Consider for simplicity a top-hat probability distribution, $p_\psi(\lambda)$. We shall be looking at two special cases $\psi = N$ and $\psi = S$ (you can think of these states as polar opposites) associated with orthogonal QWFs, N and S, respectively (that is $\langle N|S\rangle = 0$), which have a common overlapping area of an HV, λ (λ could also indicate a set of HVs). A common λ means simply that both $p_S(\lambda) \neq 0$ and $p_N(\lambda) \neq 0$ as shown in Figure 3.

First Case: Now consider the probability of obtaining a measurement of N and suppose that this probability depends only on the HV λ. We can write it as a conditional (Bayesian) probability, $M(N|\lambda)$. To obtain the total probability, $P(N|\psi)$, that is to get the probability for result N for any QWF, ψ, we must calculate $P(N|\psi) = \int M(N|\lambda)p_\psi(\lambda)d\lambda$. That is, we multiply the probability of obtaining a result for a given λ by the distribution function, $p_\psi(\lambda)$, specific to the chosen QWF, ψ, and integrate over all λ. From the Born Rule of quantum physics, $P(N|\psi) = \langle\psi|N\rangle \langle N|\psi\rangle$.

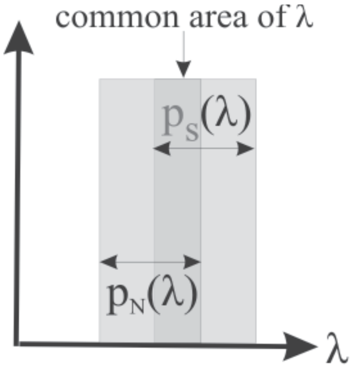

Figure 3. Conjoint top hat (overlapping) epistemic probability distributions for orthogonal quantum physics states.

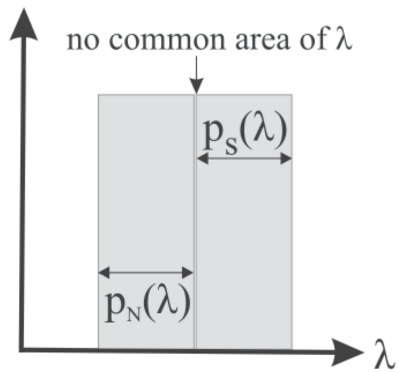

Figure 4. Disjoint epistemic probability distributions for orthogonal quantum physics states leading to ontic states |N> and |S>.

Second Case: Next consider a measurement of S which is also given by a probability, M(S|λ), which is also clearly dependent only on HV λ. Now suppose we wish to obtain the probability of getting the result, S. To obtain the total probability P(S|ψ) for getting the result S, we must have $P(S|\psi) = \int M(S|\lambda)p_\psi(\lambda)d\lambda$. And again from the Born Rule: $P(S|\psi) = \langle\psi|S\rangle\langle S|\psi\rangle$.

Now if M(S|λ) and M(N|λ) are the only probabilities for values obtained by measurements, and since there are only two such values possible, then clearly M(S|λ) + M(N|λ) = 1. There can be no other result possible and this must hold for every λ value. In plain language, specifying λ must lead to unity probability when all possible results of a measurement are taken into account with ontic variable λ specified. For example, λ could be a simple option, λ_q or λ_d, for an unseen biased coin—use a quarter or use a dime. Using a quarter, suppose $M(H|\lambda_q) = .25$ and $M(T|\lambda_q) = .75$, or using a dime suppose $M(H|\lambda_d) = .65$ and $M(T|\lambda_d) = .35$. In each HV option, dependent on the value of λ, head (H) and tail (T) are clearly orthogonal results after a toss of the coin. Again, as in the other coin example, after many such observations we could only guess the HV of the coin was a dime or a quarter because of the relative frequencies of heads to tails appearing provided we knew that just one type of coin was used each time. Otherwise we would never know which coin was used.

However, as simple as is this N or S case, it leads to a contradiction with the Born Rule of quantum physics that arises when you put $\psi = S$ in the First Case, and $\psi = N$ in the Second Case. Since S and N are orthogonal (they cannot both occur), $<S|N> = 0$. Hence in the First Case we get $<\psi|N> <N|\psi> = <S|N> <N|S> = P(N|S) = \int M(N|\lambda)p_S(\lambda)d\lambda = 0$, and in the Second Case, $<\psi|S> <S|\psi> = <N|S> <S|N> = P(S|N) = \int M(S|\lambda)p_N(\lambda)d\lambda = 0$. If these integrals are to be zero, then the integrands have to be zero for every value of λ because both $M(N|\lambda)$ and $M(S|\lambda)$ as well as $p_S(\lambda)$ and $p_N(\lambda)$ are positive functions. Therefore, in particular, these integrands have to be zero in the overlapping region. But given that both $p_S(\lambda) \neq 0$ and $p_N(\lambda) \neq 0$ in the overlapping region, that is we have overlapping distributions in λ space (see Figure 3), these results can only occur if both $M(N|\lambda) = 0$ and $M(S|\lambda) = 0$, which contradicts $M(S|\lambda) + M(N|\lambda) = 1$.

Hence for this simple orthonormal case, we cannot have both $p_S(\lambda)$ and $p_N(\lambda)$ possessing nonzero values for any common λ. In brief, they cannot have overlapping hidden variables. This means that a specification of λ leads to a unique ψ, either S or N (as in the quarter/dime example above), and we can therefore take it that $p_S(\lambda)p_N(\lambda) = 0$, so in both cases either $p_S(\lambda)$ or $p_N(\lambda)$ must be zero. PBR might call this a necessary step to proving that a QWF is an ontological function, but this proof only includes orthogonal QWFs, |N> and |S> as indicated in Figure 4. To be both necessary and sufficient one would need to show that the probability distribution $p_N(\lambda)$ for |N> and any other probability distribution $p_\psi(\lambda)$ for a QWF |ψ> cannot have any overlap even if $<N|\psi> \neq 0$.

More Complex Quantum Physics Ontology and Epistemology

In the above case we only considered orthogonal QWFs, N and S, and found them to be ontic. Can we make the argument that ψ is real in any case including nonorthogonal situations? To fully answer the query in the title of this review, we would need to look at the case when possible quantum states, α and β, are not orthogonal. One might think that since two such QWFs, |α> and |β>, do overlap, i.e. $< \beta|\alpha> \neq 0$, one might find no contradiction in having both $p_\alpha(\lambda) \neq 0$ and $p_\beta(\lambda) \neq 0$. Hence both α and β could be epistemic and still satisfy the Born Rule of quantum physics.

PBR dispel that possibility by first considering nonorthogonal states of the same simple system as above that is prepared with compass directions |N> or |E>, where $|E> = (|N> + |S>) / \sqrt{2}$, $|W> = (|N> - |S>) / \sqrt{2}$. Here we have $<N|S> = <E|W> = 0$, respectively orthogonal, but $<N|E> = 1/\sqrt{2}$, hence

N and E are not orthogonal.[4] We shall again assume that the QWF, |ψ> (either |N> or |E>), is dependent on an HV distribution $p_\psi(\lambda)$, similar to what we did in the orthogonal case above. One can recognize these "directional" states as *spinors*, i.e. spin ½ states, wherein |N> means spin up in the z direction, |S> means spin down in the z direction, |E> means spin up in the x direction, and |W> means spin down in the x direction.

The system is to be prepared in one of two ways such that one preparation produces |N> with unity probability $P(N|N) = \int M(N|\lambda)p_N(\lambda)d\lambda = 1$, arising from an epistemic $p_N(\lambda)$ distribution, while a second kind of preparation produces |E> with unity probability, $P(E|E) = \int M(E|\lambda)p_E(\lambda)d\lambda = 1$, arising from epistemic distribution $p_E(\lambda)$. The aim: <u>If</u> a specification of λ yields a specific QWF, |ψ>, orthogonal or not to any other QWF, |φ>, <u>then</u> |ψ> must be ontic and therefore an objective real "thing" "out there" independent of any observer. So, accordingly, in the case involving states |N> and |E>, in spite of the nonorthogonality of these states, the two distributions $p_N(\lambda)$ and $p_E(\lambda)$ must be disjoint, $p_N(\lambda)p_E(\lambda) = 0$, as shown in Figure 4, only substitute E for S.[5]

On the other hand, if λ lies within a region where |N> and |E> have conjoint distributions, i.e. $p_N(\lambda)$ and $p_E(\lambda)$ overlap so that $p_N(\lambda)p_E(\lambda) \neq 0$, then |ψ> cannot be ontic and must be epistemic as shown in Figure 3 (again substitute E for S).[6] In brief, an epistemic |ψ> results in a contradiction with the prediction of quantum physics just as we saw in the above N and S orthogonal case.

To clarify their argument, I will follow PBR with a slight change of notation. PBR have us consider a quantum physical situation in which two such identical, but separate, preparations $|\psi_1>$ and $|\psi_2>$ are independently made using HVs, λ_1 and λ_2, wherein both HVs lie within identical HV spaces; we have essentially two copies of the same hidden variable space. Consequently these preparations result in the uncorrelated joint quantum state $|\psi_1>|\psi_2>$, since they are produced from independent HVs. It is important to realize that PBR assume that both λ_1 and λ_2 lie within corresponding, respectively, identical HV spaces. Thus each separate space of HVs contains an identical range, $\rho \geq 0$, over which probability distributions are conjoint. Consequently each preparation produces its own corresponding HV λ_i, resulting in identical overlapping probability distributions of |N> or |E>, wherein, $p_N(\lambda_1)p_E(\lambda_1) \neq 0$ and $p_N(\lambda_2)p_E(\lambda_2) \neq 0$, provided λ_1 lies within the overlapping range, ρ, and λ_2 lies within the same

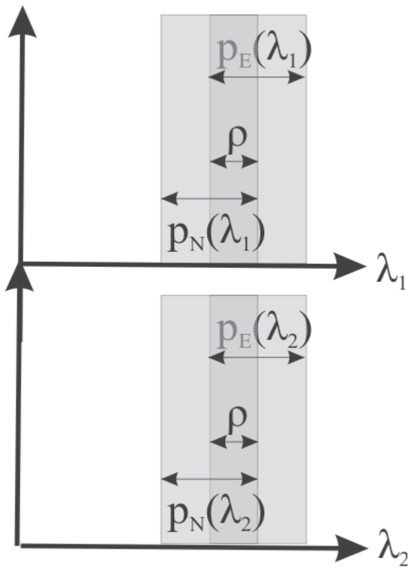

Figure 5. Conjoint top hat (overlapping) epistemic probability distributions for two identical systems with non-orthogonal quantum physics states.

correspondingly identical overlapping range, ρ, as shown in Figure 5.

That is, both systems are prepared in such a manner that we cannot uniquely determine |N> or |E>. PBR also assume the probability distribution functions, $p_N(\lambda_i)$ and $p_E(\lambda_i)$, are the same for i = 1 or 2. Since these are independent preparations, both $p_{\psi 1}(\lambda_1) \neq 0$ and $p_{\psi 2}(\lambda_2) \neq 0$ whenever λ_1 and λ_2 are each found in the same range, ρ. In Figure 5 we are essentially duplicating the scenario shown in Figure 3 for each copy.

So after preparing the joint system with both λ_1 and λ_2 in their corresponding conjoint ρ ranges, we obtain the following epistemic (possible) results for $|\psi_1>|\psi_2>$: |N>|N> or |N>|E> or |E>|N> or |E>|E>. All we need now is to specify the basis for making a measurement of the joint system. Suppose now that the two systems are brought together and measured using (projected onto) the following orthonormal entangled base states:

|1> = (|N>|S> + |S>|N>) / √2,
|2> = (|N>|W> + |S>|E>) / √2,
|3> = (|E>|S> + |W>|N>) / √2, and
|4> = (|E>|W> + |W>|E>) / √2.

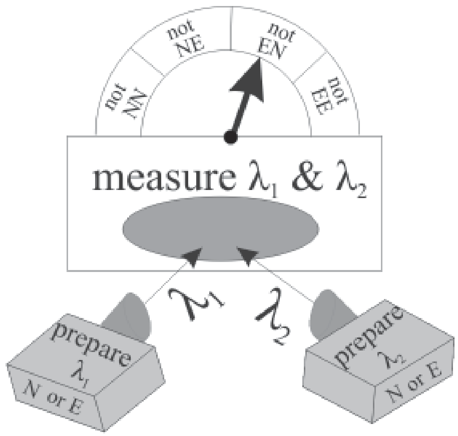

Figure 6. Experimental preparations and measurements of ontic hidden variables possibilities.

These four states are maximally entangled and orthogonal ($<i|j> = 0$, unless $i = j$, and then $<i|i> = 1$). Consequently the probability for obtaining a result, i, $P(i|\psi_1\psi_2)$, given that the joint wave function, $|\psi_1\psi_2> = |\psi_1>|\psi_2>$, can be expressed in a similar manner as for the simple case above. Following the above example and the Born Rule, we have for the joint probability, $P(i|\psi_1\psi_2) = <\psi_1\psi_2|i> <i|\psi_1\psi_2> = \iint M(i|\lambda_1,\lambda_2)p_{\psi 1}(\lambda_1)p_{\psi 2}(\lambda_2)d\lambda_1 d\lambda_2$, where the probability of obtaining a joint measurement, M, of state $|i>$ now depends on two HVs, λ_1 and λ_2, and we write it accordingly as a conditional (Bayesian) probability, $M(i|\lambda_1,\lambda_2)$. Consequently, we cover all of our four bases and find for any chosen pair of HVs, λ_1 and λ_2, $M(1|\lambda_1,\lambda_2) + M(2|\lambda_1,\lambda_2) + M(3|\lambda_1,\lambda_2) + M(4|\lambda_1,\lambda_2) = 1$. This says that the probabilities of obtaining a result for i, $1 \leq i \leq 4$, now depends on both given λ_1 and λ_2 values. Change those values and the individual $M(i|\lambda_1,\lambda_2)$ may change, as in the case of the quarter and dime; but they will always sum to unity regardless of whether or not the chosen values of λ_1 and λ_2 fall within the ranges of $\rho \geq 0$.

The question is: What are the probabilities of the results of measurement using (projecting onto) these entangled base states according to the Born Rule of quantum physics? It isn't too difficult to see that there are four cases in which we get predictions of zero probabilities—the result of a measurement will be to not find a specific result as shown in Figure 6 (based on PBR's Figure 2).

As we see next, this fact leads to a contradiction if λ_1 and λ_2 fall within the ranges of ρ, thus producing non-vanishing probability distributions. It is here where the independence and conjointness of the two individually overlapping probability distributions, $p_{\psi1}(\lambda_1)p_{\psi2}(\lambda_2) \neq 0$, play their roles.

In the first case, $P(1|NN) = <NN|1> <1|NN> = 0$, as can be seen by inspection. Therefore, $\iint M(1|\lambda_1,\lambda_2)p_N(\lambda_1)p_N(\lambda_2)d\lambda_1 d\lambda_2$ must be 0. But since λ_1 and λ_2 have non-vanishing probability distributions, $p_N(\lambda_1)p_N(\lambda_2) \neq 0$, it follows that $M(1|\lambda_1,\lambda_2) = 0$. A similar line of reasoning applies to $P(2|NE) = <NE|2> <2|NE> = 0$, where $p_N(\lambda_1)p_E(\lambda_2) \neq 0$, and for $P(3|EN) = <EN|3> <3|EN> = 0$, where $p_E(\lambda_1)p_N(\lambda_2) \neq 0$, and finally for $P(4|EE) = <EE|4> <4|EE> = 0$, where $p_E(\lambda_1)p_E(\lambda_2) \neq 0$. Remember we are assuming that $p_{\psi1}(\lambda_1)p_{\psi2}(\lambda_2) \neq 0$, corresponding to λ_1 and λ_2 falling within the ranges of ρ and these are the only cases of concern.

Therefore we would conclude for these particular values of λ_1 and λ_2, within the ranges of ρ where $p_{\psi1}(\lambda_1)p_{\psi2}(\lambda_2) \neq 0$, in each of the vanishing probabilities, $P(i|\psi_1\psi_2) = 0$, we must have $M(1|\lambda_1,\lambda_2) = 0$, $M(2|\lambda_1,\lambda_2) = 0$, $M(3|\lambda_1,\lambda_2) = 0$, and $M(4|\lambda_1,\lambda_2) = 0$, which contradicts the equation: $M(1|\lambda_1,\lambda_2) + M(2|\lambda_1,\lambda_2) + M(3|\lambda_1,\lambda_2) + M(4|\lambda_1,\lambda_2) = 1$, which is valid for all values of λ_1 and λ_2. The only way out of the contradiction is, of course, to deny that the non-vanishing probability distributions, where λ_1 and λ_2 are within the supported "overlapping" ranges of values of ρ, $p_{\psi1}(\lambda_1)p_{\psi2}(\lambda_2) \neq 0$, can ever occur. Thus $P(1|NN) = 0$ implies that $p_N(\lambda_1)p_N(\lambda_2) = 0$, $P(2|NE) = 0$ implies that $p_N(\lambda_1)p_E(\lambda_2) = 0$, $P(3|EN) = 0$ implies that $p_E(\lambda_1)p_N(\lambda_2) = 0$, and $P(4|EE) = 0$ implies that $p_E(\lambda_1)p_E(\lambda_2) = 0$. In each case it's necessary and sufficient that only one of the pairs of $p_{\psi i}(\lambda_i)$s need vanish to rule out any overlap and thus rule in that all such ψ_is are ontological. Having either $p_{\psi i}(\lambda_i)$vanish means $p_{\psi1}(\lambda_1)p_{\psi2}(\lambda_2) = 0$, and consequently since both ψ_1 and ψ_2 are either N or E then the condition $p_{\psi1}(\lambda_1)p_{\psi2}(\lambda_2) \neq 0$ is equally ruled out for each ψ_i. Thus for any pair of nonorthogonal ψ_is, the Born Rule of quantum physics cannot be satisfied, if their respective HV probabilities overlap.

Simple Illustration of the BPR Theorem for Two Non-Orthogonal States

Of course, it could be that for most values of λ_1 and λ_2, outside the range of ρ, or indeed if $\rho = 0$, the condition $p_{\psi1}(\lambda_1)p_{\psi2}(\lambda_2) = 0$ need not arise to have $P(i|\psi_1\psi_2) = 0$, and for these cases no contradiction arises. To further clarify the argument consider Figure 7, where I show a possible set of conditional

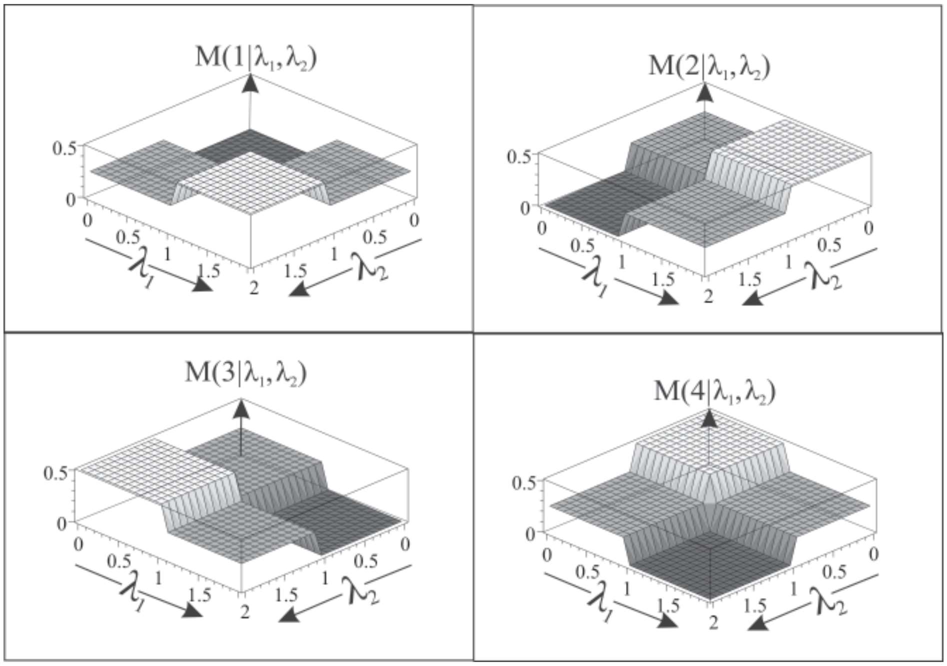

Figure 7. Three dimensional views of quilted, stepped, conditional measurement probabilities, $M(i|\lambda_1,\lambda_2)$, consistent with disjoint top hat probability distributions for two identical systems with non-orthogonal quantum physics states.

measurement probability distributions, $M(i|\lambda_1,\lambda_2)$, consistent with nonoverlapping top-hat probability distributions shown in Figure 5 with $\rho = 0$. Each conditional measurement probability distribution consists of a quilt of four patches with $M(i|\lambda_1,\lambda_2)$ being constant in each patch and $i \in (1,4)$. The darkest patch has $M(i|\lambda_1,\lambda_2) = 0$, the light grey patches have $M(i|\lambda_1,\lambda_2) = .25$, and the nearly white patch has $M(i|\lambda_1,\lambda_2) = .50$. One can see by inspection that $M(1|\lambda_1,\lambda_2) + M(2|\lambda_1,\lambda_2) + M(3|\lambda_1,\lambda_2) + M(4|\lambda_1,\lambda_2) = 1$ for any pair of values, (λ_1,λ_2), in the quilt. So long as $\rho = 0$, we never see any contradiction arising with the Born Rule because the disjoint probability distributions, $p_{\psi 1}(\lambda_1)$and $p_{\psi 2}(\lambda_2)$, are consistently defined within the same boundaries as the quilted measurement probabilities, $M(i|\lambda_1,\lambda_2)$. It is only when $p_{\psi 1}(\lambda_1)$ and $p_{\psi 2}(\lambda_2)$ exceed those quilted boundaries that contradictions arise as indicated next.

If we have $\rho > 0$, then these measurement probabilities, $M(i|\lambda_1,\lambda_2)$, lead to contradiction with the Born Rule. To see this in each of the four cases, let us again consider our conjoint top-hat probability distributions, as shown in Figure 5 such that, $p_N(\lambda_1) = p_N(\lambda_2) = 1/(1+\rho/2)$ in the ρ-extended range, when

$0 \leq \lambda_1 \leq (1+\rho/2)$ and $0 \leq \lambda_2 \leq (1+\rho/2)$, respectively, and 0 elsewhere. And similarly for $p_E(\lambda_1) = p_E(\lambda_2) = 1/(1+\rho/2)$ in the ρ-extended ranges, $(1-\rho/2) \leq \lambda_1 \leq 2$ and $(1-\rho/2) \leq \lambda_2 \leq 2$, respectively, and 0 elsewhere. Consequently we have the normalized probabilities, $\int p_N(\lambda_i)d\lambda_i = \int p_E(\lambda_i)d\lambda_i = 1$, for i = 1,2.

Case 1. Let us now examine the first case where $P(1|NN) = \langle NN|1\rangle \langle 1|NN\rangle = \iint M(1|\lambda_1,\lambda_2)p_N(\lambda_1)p_N(\lambda_2)d\lambda_1 d\lambda_2 = 0$, according to the Born Rule. There is no problem for $0 \leq \lambda_1 \leq 1$ and $0 \leq \lambda_2 \leq 1$; we simply have on this patch of the λ-quilt, $M(1|\lambda_1,\lambda_2) = 0$. However, in the overlapping ranges, $1 < \lambda_1 \leq (1+\rho/2)$ and $1 < \lambda_2 \leq (1+\rho/2)$, $M(1|\lambda_1,\lambda_2) = .5$, and consequently $P(1|NN) = \rho^2/[8(1+\rho/2)^2] \neq 0$, in contradiction of the Born Rule.

Case 2. A similar line of reasoning applies for $P(2|NE) = \langle NE|2\rangle \langle 2|NE\rangle = \iint M(2|\lambda_1,\lambda_2)p_N(\lambda_1)p_E(\lambda_2)d\lambda_1 d\lambda_2 = 0$, according to the Born Rule. Here we again have no problem for $0 \leq \lambda_1 \leq 1$ and $1 \leq \lambda_2 \leq 2$. On this patch of the λ-quilt, $M(2|\lambda_1,\lambda_2) = 0$. However, for $1 \leq \lambda_1 \leq (1+\rho/2)$ and $(1-\rho/2) \leq \lambda_2 \leq 1$, we have $M(2|\lambda_1,\lambda_2) = .5$ and consequently $P(2|NE) = \rho^2/[8(1+\rho/2)^2] \neq 0$, as in the first case, in contradiction of the Born Rule.

Case 3. A similar line of reasoning applies for $P(3|EN) = \langle EN|3\rangle \langle 3|EN\rangle = \iint M(3|\lambda_1,\lambda_2)p_E(\lambda_1)p_N(\lambda_2)d\lambda_1 d\lambda_2 = 0$, according to the Born Rule. Here we again have no problem for $1 \leq \lambda_1 \leq 2$ and $0 \leq \lambda_2 \leq 1$. On this patch of the λ-quilt, $M(3|\lambda_1,\lambda_2) = 0$. However for $(1-\rho/2) \leq \lambda_1 \leq 1$ and $1 \leq \lambda_2 \leq (1+\rho/2)$, we have $M(3|\lambda_1,\lambda_2) = .5$ and consequently $P(3|EN) = \rho^2/[8(1+\rho/2)^2] \neq 0$, as in the first case, in contradiction of the Born Rule.

Case 4. A similar line of reasoning applies for $P(4|EE) = \langle EE|4\rangle \langle 4|EE\rangle = \iint M(4|\lambda_1,\lambda_2)p_E(\lambda_1)p_E(\lambda_2)d\lambda_1 d\lambda_2 = 0$, according to the Born Rule. Here we again have no problem for $1 \leq \lambda_1 \leq 2$ and $1 \leq \lambda_2 \leq 2$. On this patch of the λ-quilt, $M(4|\lambda_1,\lambda_2 2) = 0$. However for $(1-\rho/2) \leq \lambda_1 \leq 1$ and $(1-\rho/2) \leq \lambda_2 \leq 1$,we have $M(4|\lambda_1,\lambda_2) = .5$ and consequently $P(4|EE) = \rho^2/[8(1+\rho/2)^2] \neq 0$, as in the first case, in contradiction of the Born Rule.

Of course, in each case, in the limit where $\rho \to 0$, no contradiction arises and the correct results for the measurement probabilities are obtained. Thus, for example, from the top right-hand corner of Figure 7 dealing with measurements projected onto the |2> state we find:

$P(2|NE) = \langle NE|2\rangle \langle 2|NE\rangle = \iint M(2|\lambda_1,\lambda_2)p_N(\lambda_1)p_E(\lambda_2)d\lambda_1 d\lambda_2 = 0$,
$P(2|NN) = \langle NN|2\rangle \langle 2|NN\rangle = \iint M(2|\lambda_1,\lambda_2)p_N(\lambda_1)p_N(\lambda_2)d\lambda_1 d\lambda_2 = .25$,
$P(2|EN) = \langle EN|2\rangle \langle 2|EN\rangle = \iint M(2|\lambda_1,\lambda_2)p_E(\lambda_1 1)p_N(\lambda_2)d\lambda_1 d\lambda_2 = .50$, and
$P(2|EE) = \langle EE|2\rangle \langle 2|EE\rangle = \iint M(2|\lambda_1,\lambda_2)p_E(\lambda_1)p_E(\lambda_2)d\lambda_1 d\lambda_2 = .25$,

all consistent with the Born Rule leading to unity probability when summed. Similar results follow for the other measurements projected onto the |i> state, with i = 1, 3, and 4.

Discussion

To prove or disprove whether or not any general QWF |α> is ontic is quite an accomplishment even for a limited HV, but a clever approach as taken by PBR. To establish that a given |α> is ontic, you have to construct an argument showing that for any other QWF, |β>, even when <β|α> ≠ 0, it is always possible to find such a contradiction as shown above. They use n identically prepared and uncorrelated independent QWFs (I looked at n = 2) generating a QWF, |Ψ> = |ψ$_1$>|ψ$_2$> . . . |ψ$_n$>, where each QWF is either |α> or |β>. |Ψ> is projected onto an entangled QWF measuring device (a combination of various gates and other devices used in quantum computers called a measurement circuit) that jointly measures the n systems in such a manner that there is always at least one of the 2^n QWFs predicted with zero probability. Indeed this is a very clever idea as one can nearly always show[7] that |Ψ>, being a product of independent QWFs, must consist of independent ontic states.

On the other hand, if a measurement of a state with zero probability ever occurs (e.g., corresponding to an EN measurement when a not-EN state was prepared, as indicated in Figure 6), indicating a violation of the predicted quantum probabilities, does that indicate Einstein was right after all and quantum physics is ontologically incomplete?[8]

Could this be proven experimentally? All one would need to do is show that the condition of never finding a zero probability case in any the 2^n possible cases would possibly do it. Suppose that indeed one were to find all (measurement) projections onto such entangled base states devices never occurring with zero probability.[9] According to PBR the epistemic nature of QWFs in violation of quantum physics would be established. Einstein would emerge victorious and we would need a new physics beyond quantum physics.

In summary we have a logical proof here: For two or more QWFs the Born Rule (TBR) implies disjoint HV probability distributions (DPD), TBR → DPD. However DPD does not necessarily imply the Born Rule ~(DPD → TBR). They are not equivalent. The important statement of PBR is that conjoint probability distributions (CPD) violate the Born Rule, (CPD → ~TBR). That means CPD make the quantum state unknown and hence epistemological. CPD mean the quantum state is not fixed by a determination of the HV. A given HV will produce more than one quantum state possibility—hence the quantum state is epistemological. Since ~CPD is the same as DPD and CPD implies a negation of the Born Rule, CPD → ~TBR; reversing the logic we get TBR → ~CPD so TBR → DPD.

Let me add a few more comments of my own here. I believe that until the ontology/epistemology issue is fully resolved (although readers may believe it already resolved after reading this review), we still have the "measurement problem" that stimulated such considerations as given by PBR, Bell, Bohm, and many others. We also still have the nonlocality issue to deal with. Perhaps PBR can resolve this issue. Ontologically speaking, what does it mean to have nonlocal influences? What does it mean to have an observer effect (collapse of the QWF)? Does the PBR solution resolve these problems?

Consider the effect of observation on an ontic QWF. Does a human being alter the QWF simply by making an observation? If the QWF is ontic then we have a real observer effect—observation (including nonlocal) indeed alters the QWF and therefore reality. That would mean that mind is inextricably tied into matter; they are truly entangled, and such a finding could lead to breaking discoveries in the study of consciousness. On the other hand, if the QWF proves to be epistemic in violation of the Born Probability Rule, observation is simply the usage of the Bayesian approach to probabilities wherein new information simply changes what we know, but leaves reality unscathed—at least what we mean by ontic reality. I hope that PBR and others continue this line of research. The next frontier may indeed not be space but will be the mind.

Notes

[1] Indeed Einstein did make this conclusion based on the EPR argument. However, it is not a conclusion of Bell's theorem and certainly not Einstein's conclusion based on Bell's work because he was dead at the time. In fact, Bell's theorem rather stymies this line of argument, since it says that you will still have nonlocal influences even if the wave function is epistemic, so this move does not solve the problem of nonlocality.

[2] One may need to allow for the fact that measurements might be fundamentally noisy or stochastic and demand only that HVs specify probabilities for any measurement outcome.

[3] In this SHO example (with m = ½ and k = 2), assuming t = 0, the spring is stretched to a distance, √E, we get x = (√E)cos(2t) and p = (−√E)sin(2t). The point in the phase plane rotates clockwise around the circle completing the cycle in the period of π. The probability density is simply a constant, dP/dt = 1/π, for all such circles regardless of the energy. Indeed that's why spring clocks work.

[4] This sounds peculiar since clearly the directions are perpendicular. However, perpendicular in space does not necessarily mean the same thing as orthogonal in quantum physics. For those who know a little quantum physics: Two quantum states α and β are orthogonal if and only if $\langle\alpha|\beta\rangle = 0$.

[5] That is, there is no overlap of these probability distributions, so we have $p_N(\lambda)p_E(\lambda) = 0$. So this means either $p_N(\lambda) = 0$ or $p_E(\lambda) = 0$ for all λ.

[6] Here there is an overlap, so $p_N(\lambda)p_E(\lambda) \neq 0$. So that means both $p_N(\lambda) \neq 0$ and $p_E(\lambda) \neq 0$ for λ within the overlap region.

[7] PBR also carry out an error analysis to complete their proof.

[8] Such a violation would tell us that it is possible, i.e. not in conflict with experimental results, that the wave function is epistemic.

[9] Matt Leifer in an email to me pointed out that from any epistemic HV theory, you can always construct one that is ontological and gives exactly the same predictions. Such an argument is given in M. Schlosshauer and A. Fine, "Implications of the Pusey–Barrett–Rudolph no-go theorem," http://arxiv.org/abs/1203.4779. Consequently Leifer doesn't think it is possible to establish that the QWF is epistemic purely by experiment.

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References

Jaynes, E. T. (1989). Clearing up Mysteries. In J Skilling, Editor, Proceedings volume *Maximum Entropy and Bayesian Methods*, Dordrecht, Holland: Kluwer Academic Publishing, pp. 1–27.

Pusey, M. S., Barrett, J., & Rudolph, T. (2012). The quantum state cannot be interpreted statistically. http://arxiv.org/abs/1111.3328v1

Reich, E. S. (2012). A boost for quantum reality. *Nature*, *485*(7397), 10 May, 157–158. doi:10.1038/485157a